Frontier Topics in Empirical Economics: Week 2
Non-parametric Method

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Non-parametric Method: Introduction

- Common Parametric Models
  Linear Model: $y = X'\beta + e$, $e \sim N(0, \sigma^2)$;
  Probit/Logit Model: $P(y|X) = G(X\beta)$ where G is a nonlinear function
- Explicit Parametric Structure for Distribution
- Common Estimator
  OLS, MLE, Nonlinear LS, Efficient GMM etc.
- Key Properties of the Estimator
  Consistency, BLUE, Asymptotic Efficiency etc.

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- In linear regression, we have to assume that CEF is linear
- Why linear? Simple? Why not $y = x^3 \cdot lnx + e$?
- What if linear specification is wrong?
- Everything collapses. No data can save.
- It becomes only a linear approximation
- For example, if true model is Logit, but not linear regression

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- Potential Outcome Model is intrinsically NON-parametric!!!
- There are many other statistical modeling methods
- Non-parametric, semi-parametric
- To understand tools beyond linear regression!

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- Give up the "parametric" model like linear regression
- Do not assume that CEF is linear
- Go back to the original question to estimate $E(y_i|x_i)$ without imposing any functional form assumption

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- Notation: x_i, y_i denotes random variable; X_i, Y_i denotes realizations; x, y denotes random variables or some value of the random variables
- Realizations are given (sample), they are NOT random in our context
  $\int x \sum_i^n X_i dx = \sum_i^n X_i \int x dx$

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Non-parametric Method: Kernel Regression

- Let's consider the first non-parametric method: Kernel regression
- It is super intuitive and interesting
- Instead of assuming $E(y_i|x_i) = x_i'\beta$, we consider this CEF point by point
- That is, estimate $E(y_i|x_i)$ for each possible point of $x_i = x$

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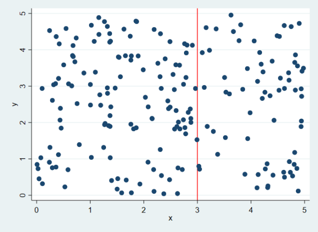
Step 1: Estimating a cumulative density

- Consider estimating a cumulative density function (CDF)



- What is the CDF at $x = 3$? $\hat{F}(x = 3) = ?$
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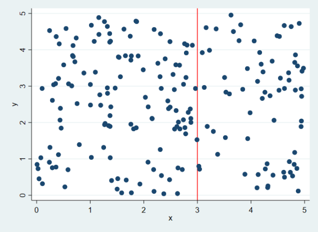

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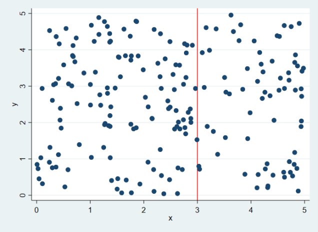

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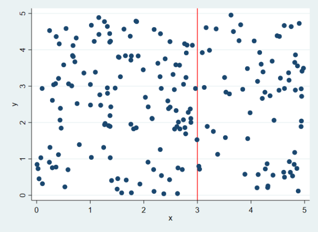

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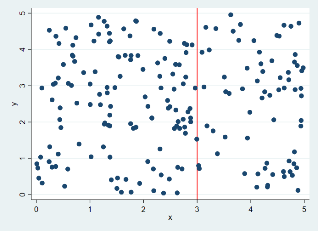

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Non-parametric Method: Kernel Regression

- Just count how many points lie on the left to the red line:

$$\hat{F}(x = 3) = \frac{1}{n} \sum \mathbf{1}(X_i \leq 3)$$

- In general, we have an estimation of $F(x)$ as:

$$F(x) = P(X \leq x) \Rightarrow \hat{F}(x) = \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}(X_i \leq x)$$

- The proportion of points (realizations) that are smaller than x

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Non-parametric Method: Kernel Regression

Step 2: Estimating a probability density

- Consider estimating a probability density function (PDF)
- PDF represents a marginal increase in CDF at some point (derivative)

$$f(x) = \frac{dF(x)}{dx} = \lim_{h \to 0} \frac{F(x+h) - F(x-h)}{2h}$$

$$\hat{f}(x) = \frac{\hat{F}(x+h) - \hat{F}(x-h)}{2h}$$

- Changes of $F(x)$ in a very small interval (with length $2h$)
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- Then we can write the probability density $f(x)$ at some value x as:

$$\hat{f}(x) = \frac{1}{2h}\left[\frac{1}{n}\sum_{i=1}^{n}\mathbf{1}(X_i \le x+h) - \frac{1}{n}\sum_{i=1}^{n}\mathbf{1}(X_i \le x-h)\right]$$

$$= \frac{1}{n}\sum_{i=1}^{n}\frac{1}{2h}\mathbf{1}(x-h \le X_i \le x+h)$$

- How to interpret this?
- We count the number of obs within a small interval around x, dividing by the length and the total number of obs
- $\sum_{i=1}^{n}\frac{1}{2h}\mathbf{1}(x-h \le X_i \le x+h)$ is the number of obs per unit length
- When n is large, we can choose very small h

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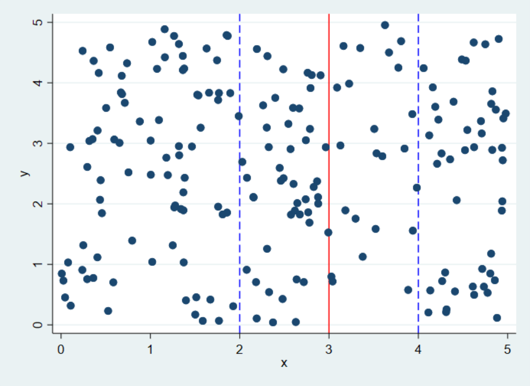

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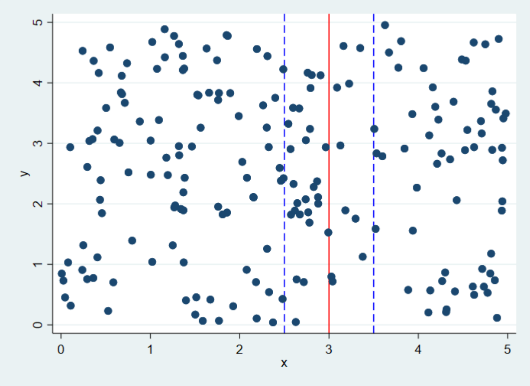

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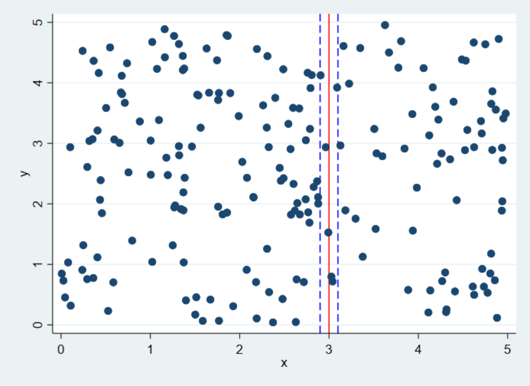

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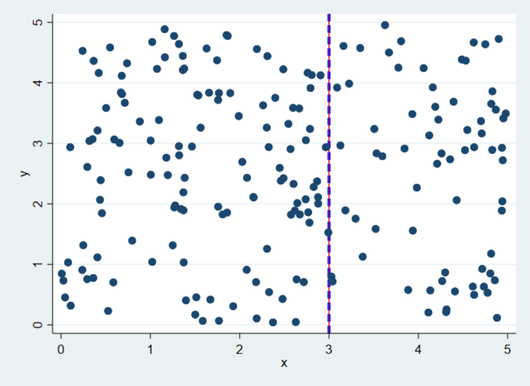

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- Define $k(v) = \frac{1}{2}\mathbf{1}(|v| \leq 1)$. Then we have:

$$\hat{f}(x) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{h}k(\frac{X_i - x}{h})$$

- We call $k(v)$ a uniform kernel function
- This $\hat{f}(x)$ is a kernel estimator of the PDF (uniform kernel)
- Kernel is weight!
- There can be other kinds of kernel functions, when we assign different weights to different observations

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$$\hat{f}(x) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{h}k(\frac{X_i - x}{h})$$

- We call $k(v)$ a uniform kernel function
- This $\hat{f}(x)$ is a kernel estimator of the PDF (uniform kernel)
- Kernel is weight!
- There can be other kinds of kernel functions, when we assign different weights to different observations

Non-parametric Method: Kernel Regression

- A function can be used as a kernel if
  - $k(v)$ is integrated to 1
  - $k(v)$ is symmetric with $k(v) = k(-v)$
- The weights sum to one; The weights are symmetric to the left and to the right
- Triangular Kernel: $k(v) = (1 - |v|)\mathbf{1}(|v| \leq 1)$
- Epanechnikov Kernel: $k(v) = \frac{3}{4}(1 - v^2)\mathbf{1}(|v| \leq 1)$
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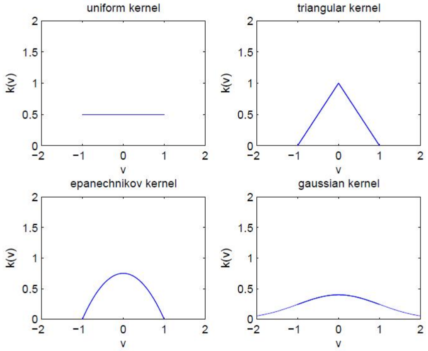

Figure 1: Various Kernels

Non-parametric Method: Kernel Regression

- For multivariate case, let $v = (v_1, v_2, \cdots, v_q)$.
- Define product kernel: $K(v) = k(v_1)k(v_2)\cdots, k(v_q)$.
- The estimator becomes:

$$\hat{f}(x) = \frac{1}{nh_1 h_2 \cdots, h_q} \sum_i K(\frac{X_i - x}{h})$$

- $h = (h_1, h_2, \cdots, h_q)$

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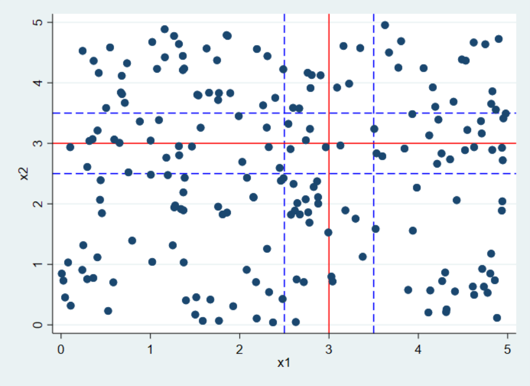

Non-parametric Method: Kernel Regression

Step 3: Estimating a CEF

- Finally, let's see how to estimate a CEF using kernel method
- Not like linear regression, we estimate the CEF point by point
- Assume that we have CEF:

$$Y = g(X) + u$$
$$E[Y|X] = g(X)$$

- u has a conditional variance $Var(u|X) = \sigma^2$

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Step 3: Estimating a CEF

- Based on the CDF and PDF we've got, we have Nadaraya-Watson Estimator (N-W) for CEF as follows:

$$\hat{g}(x) = \sum_{i=1}^{n} Y_i K_h(X_i - x) \quad where \quad K_h(X_i - x) = \frac{K(\frac{X_i - x}{h})}{\sum_{i=1}^{n} K(\frac{X_i - x}{h})}$$

- Intuition: The conditional Expectation of Y given X=x is estimated as a weighted average of observed Y_i closely around x (within the range of bandwidth h).
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Homework:

- 1. Derive NW Estimator from the kernel estimator of CDF and PDF. This can be a little bit hard. You can refer to Notes from Carol (or Hansen's book) for help.
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- We have $g(x) = E(Y|X)$ as CEF and $f(x)$ as density for x

Theorem (Asymptotics for N-W Estimator)

Under some regularity conditions, as $n \to \infty, h_s \to 0 (s = 1, \ldots, q)$, $nh_1 \ldots h_q \to \infty$ and $nh_1 \ldots h_q \sum_{s=1}^{q} h_s^6 \to 0$, we have:

$$\sqrt{nh_1 \ldots h_q}(\hat{g}(x) - g(x) - \sum_{s=1}^{q} h_s^2 B_s(x)) \xrightarrow{d} N(0, \frac{\sigma^2(x)}{f(x)}(\int k(v)^2 dv)^q)$$

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Non-parametric Method: Kernel Regression

Asymptotic Bias= $\sum_{s=1}^{q} h_s^2 \frac{\int v^2 k(v)dv}{2f(x)}[2\frac{\partial f(x)}{\partial x_s}\frac{\partial g(x)}{\partial x_s} + f(x)\frac{\partial^2 g(x)}{\partial x_s^2}]$

Asymptotic Variance= $\frac{1}{nh_1 \dots h_q}\frac{\sigma^2(x)}{f(x)}(\int k(v)^2 dv)^q$

- (1) $h_s \uparrow \Longrightarrow Bias \uparrow, Variance \downarrow$
  $\therefore$ we have trade-off in choosing kernel bandwidth.
- (2) $q \uparrow \Longrightarrow Variance \uparrow \ exponentially$
  We call this "Curse of Dimensionality".
- (3) Kernel more concentrated $\Rightarrow Bias \downarrow (\int v^2 k(v)dv), Variance \uparrow (\int k(v)^2 dv)$
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- (4) Slope Effect and Curvature Effect on bias: $\frac{\partial f(x)}{\partial x_s}\frac{\partial g(x)}{\partial x_s}, \frac{\partial^2 g(x)}{\partial x_s^2}$
- (5) $f(x) \uparrow \Longrightarrow Bias \downarrow, Variance \downarrow$ (more observations)

Non-parametric Method: Local Polynomial

- Another widely used kernel-based method is local polynomial
- In linear regression, we use a global linear function to fit data
- In local polynomial, we use piece-wise polynomial (linear) function to fit data interval by interval

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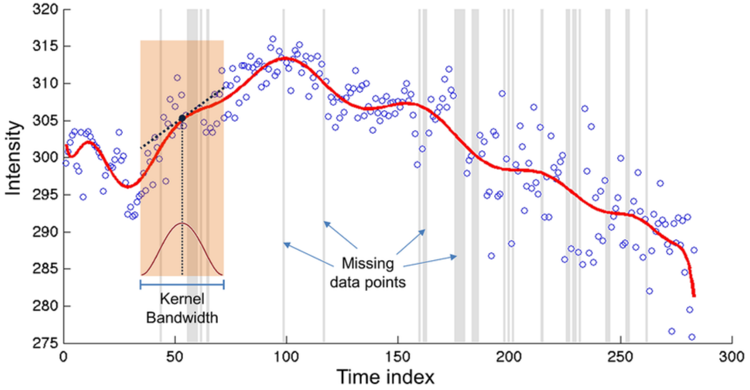

For some $X = x$, we fit $g(x)$ by choosing samples very close to x. Then we fit a polynomial for these observations. (Here, linear)

Non-parametric Method: Local Polynomial

- For $g(x)$, we solve the following optimization problem at each point x:

$$\min_{b_0, b_1, \cdots, b_p} \sum_{i=1}^{n} k(\frac{X_i - x}{h})(Y_i - b_0 - b_1(X_i - x) - b_2(X_i - x)^2 - \cdots - b_p(X_i - x)^p)^2$$

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- When $p = 2$, we call it local quadratic regression

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- Both kernel and local polynomial regressions are Kernel-based methods
- There are three disadvantages of this method:
  - Computational burden is large
  - Hard to include information or restriction over functional form
  - Requirement of large sample
- Series-based methods alleviate these problems

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$$Y = g(X) + u$$
$$g(X) = E(Y|X)$$

- We expand the CEF by Taylor Series at zero:

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- We can use OLS to estimate this polynomial
- The vector of $\{p_0, p_1, p_2, ..., p_K\}$ is called "basis"
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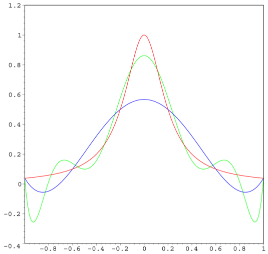
- Red: original function; Blue: fifth-order poly; Green: ninth-order poly



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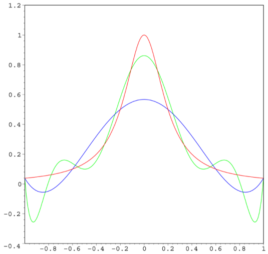

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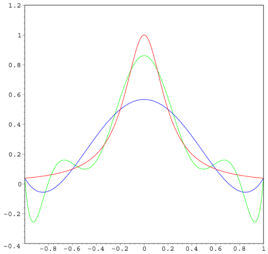

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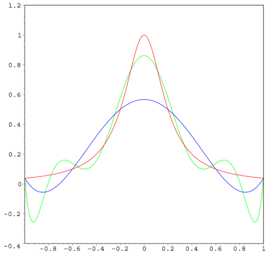

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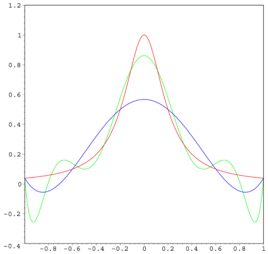

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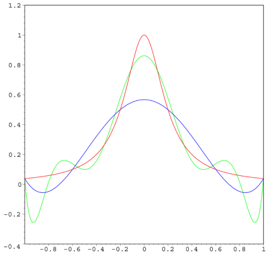

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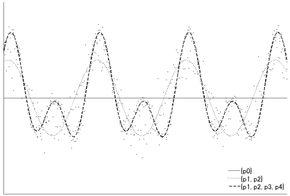

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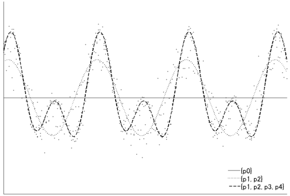

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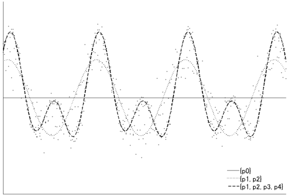

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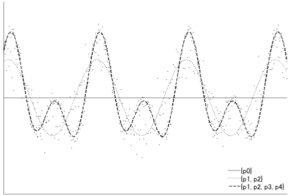

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- Such as Spline basis and Wavelet basis
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- Non-parametric model is so general that we do not impose any structure
- Totally data driven, no prior information
- Convergence rate is low, variance is high, requirement for data is high
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- Semi-parametric model

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- One of the most popular semi-parametric models

$$Y = X'\beta + g(Z) + u, \quad E(u|X, Z) = 0, Var(u|X, Z) = \sigma^2$$

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- Then we have estimators for $Y - E(Y|Z)$ and $X - E(X|Z)$
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- In the second step, we subtract $X'\beta$ from Y:

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- Question: How to estimate the variance of $\hat{g}(Z)$?
- Can we use the variance from the non-parametric regression directly?
- No! Because $Y - X'\beta$ is also estimated
- It contains more uncertainty from the first step
- We need bootstrap

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Non-parametric Method: Bootstrap

- Bootstrap is a non-parametric method for inference
- Instead of deriving the closed-form equation of variance
- We use simulation to estimate it
- Random sampling with replacement

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- Step 1: Given full sample with size n, draw R new samples of size n, with replacement. Index each new sample by r
- Step 2: Calculate the simulated variance of $\hat{g}(x)$ by:
  $\hat{V}(x) = \frac{1}{R-1} \sum_{r=1}^{R} [\hat{g}_r(x) - \hat{g}(x)]^2$
- Step 3: Use $\hat{V}(x)$ to calculate confidence intervals and implement statistical tests
- We call this bootstrapped variance

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- But using bootstrapped variance to construct confidence interval is a poor choice
- It relies on asymptotic normality, which is not accurate in finite sample
- A better chioce is "percentile interval"
- First, we stack the sample of bootstrap estimates $\{\hat{\beta}^1, \hat{\beta}^2, ..., \hat{\beta}^R\}$
- We have an empirical distribution of $\hat{\beta}^r$
- The bootstrap $100(1-\alpha)\%$ confidence interval is then: $[q^*_{\alpha/2}, q^*_{1-\alpha/2}]$
- q^* is the quantile of this empirical distribution

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Non-parametric Method: Application

- Anything related to estimation of CEF
- Potential outcome framework is non-parametric
  Causal inference highly depends on non-parametric techniques
- Non-parametric inference in complicated models (Bootstrap)
- If you focus on prediction and fit, but not the structure behind it
  Predict stock price, machine learning, RDD fitting
- We will show these in the following lectures

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Non-parametric Method: Application

- Paper report
  Dube et al. (2020) Monopsony in Online Labor Markets

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Final Conclusion

- There are statistical modeling methods other than Linear regression
- Non-parametric methods impose no prior structure, totally data-driven
  - Kernel-based methods: N-W estimator, Local polynomials
  - Series-based methods: Polynomial, Fourier, Spline, Wavelet
- They are very useful when you want to do prediction, or when you want to implement causal inference in a complicated context
- However, they have weaknesses: Not always better to make model more flexible
  - Hard to incorporate restrictions
  - Require large sample size to have accurate estimation
- We will discuss more about it next week
- A semi-parametric model is between non-parametric and parametric

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