

# Frontier Topics in Empirical Economics: Week 4

## Directed Acyclic Graph

Zibin Huang<sup>1</sup>

<sup>1</sup>College of Business, Shanghai University of Finance and Economics

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# Introduction

- Causal inference is the central topic of applied economics
- We almost solely focus on potential outcome framework in Economics
- This framework is proposed by Donald Rubin (Imbens and Rubin, 2015; Rubin, 1974) and sometimes called "Rubin Causal Model"

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- Is this the only statistical framework dealing with causal inference issue?  
Of course NOT.
- Graphical Model is another important method (Pearl, 2009)
- This is a method highly related to computer science and AI
- Nobel Prize: AI is the future of all sciences!! LOL

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- Today we are going to learn this new framework
- How it can be applied to economic research is still a *very very open question*
- Imbens wrote an interesting and critical paper on it  
Imbens (2020) Potential Outcome and Directed Acyclic Graph Approaches to Causality: Relevance for Empirical Practice in Economics

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- Introduce the graphical model and the DAG framework
- Discuss the possible usage of DAG for economists: Pros and Cons
- Compare DAG and PO framework: why PO is still more popular
- An example of using DAG: Pinto (2015)
- Conclusion: How can DAG help applied economics research (open question)

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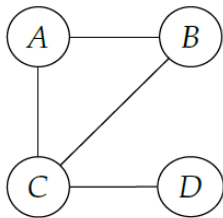
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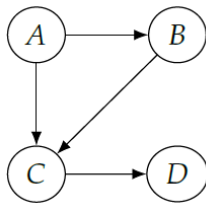
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# DAG Approach: Graph

- Graph is a collection of *nodes* and *edges* that connect the nodes.
- Two nodes are called *adjacent* if they are connected by an edge.
- A directed graph's edges go out of a *parent* into a *child*.
- A *path* is any sequence of adjacent nodes, regardless of the direction of the edges. A *directed path* is a path that consists of directed edges that are all directed in the same direction.



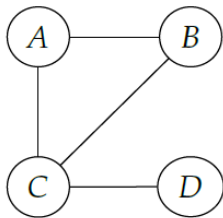
(a) Undirected Graph



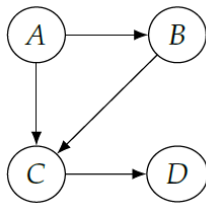
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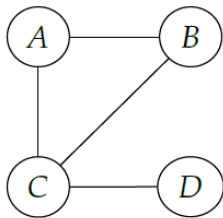
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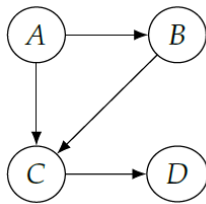
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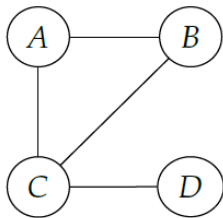
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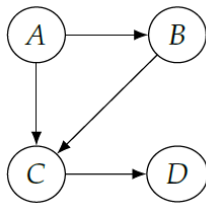
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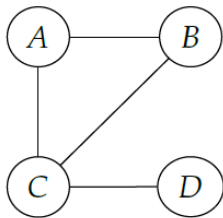
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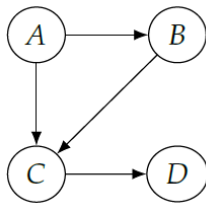
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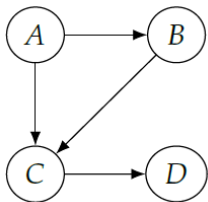


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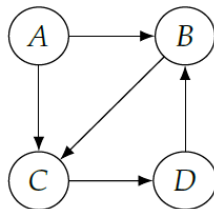


# DAG Approach: Graph

- If there is a directed path that starts at node  $X$  and ends at node  $Y$ , then  $X$  is an ancestor of  $Y$ , and  $Y$  is a descendant of  $X$ .
- If there is no cycle in a directed graph, the graph is called a *directed acyclic graph* (DAG)



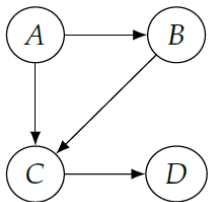
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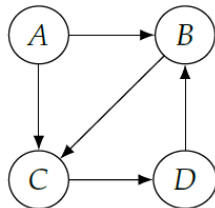
(l) Cycle

# DAG Approach: Graph

- If there is a directed path that starts at node X and ends at node Y, then X is an ancestor of Y, and Y is a descendant of X.
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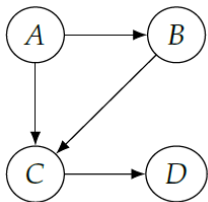
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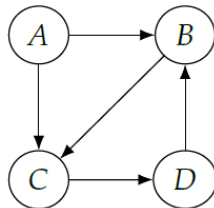
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- The first step is to connect graphs to statistical relations: Bayesian Networks
- A Bayesian network is a probabilistic graphical model that represents a set of variables and their conditional dependencies via a DAG

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# DAG Approach: Bayesian Networks

- For **any PDF**, a Bayesian factorization can be expressed as:

$$P(x_1, x_2, \dots, x_n) = P(x_1) \prod_{i=2}^n P(x_i | x_{i-1}, \dots, x_1) \quad (1)$$

- Example:  $P(x_1, x_2, x_3) = P(x_1)P(x_2|x_1)P(x_3|x_2, x_1)$
- This is like a chain
- We can simplify the model if we assume some dependency structure, e.g.  
 $P(x_3|x_2, x_1) = P(x_3|x_2)$  if  $x_1 \perp x_3|x_2$
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# DAG Approach: Bayesian Networks

- Bayesian factorization can be applied to any joint distribution of  $(X_1, X_2, \dots, X_n)$
- With the set of the dependency assumptions, we are giving the joint distribution a structure
- We can use a graph to represent this assumed dependency structure, system of probabilistic relations
- A one-to-one mapping between graph  $G$  and probabilistic relations  $P$

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## Assumption (Minimality Assumption)

1. *Given its parents in the DAG, a node  $X$  is independent of all its non-descendants (Local Markov Assumption);*
2. *Adjacent nodes in the DAG are dependent (Minimal independence).*

## Definition (Bayesian Network Factorization)

*Given a probability distribution  $P$  and a DAG  $G$  satisfying "Minimality Assumption",  $P$  factorizes according to  $G$  by*

$$P(x_1, x_2, \dots, x_n) = P(x_1) \prod_i P(x_i | pa_i)$$

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- Local Markov means that the dependence structure is "local" and "Markov"
- Minimal independence means that there is no more independence outside the network showed in the graph
- Bayesian Factorization means that: If  $P$  has a causal structure as shown in  $G$ 
  - $X_i$  only depends on parents  $Pa_i$  in the graph
  - We can do Bayesian network factorization for  $P$  w.r.t.  $G$
- We call " $G$  represents  $P$ ", " $G$  and  $P$  are compatible", " $P$  is Markov relative to  $G$ "

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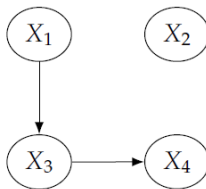
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# DAG Approach: Bayesian Networks

- Let's see a simple example
- Assume that we have four variables  $x_1, x_2, x_3, x_4$
- A full decomposition is:

$$P(x_1, x_2, x_3, x_4) = P(x_1)P(x_2|x_1)P(x_3|x_2, x_1)P(x_4|x_3, x_2, x_1) \quad (2)$$

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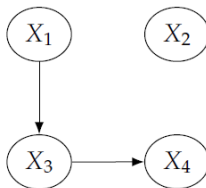


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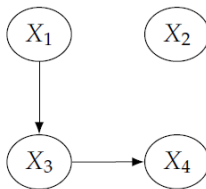


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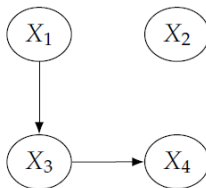


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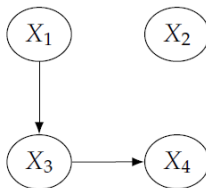


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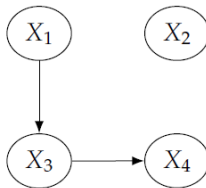


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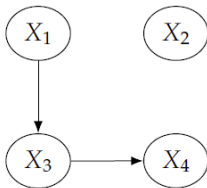


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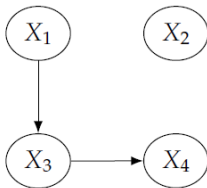


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# DAG Approach: Causal Graphs

- Up until now, we consider only statistical dependencies
- What about those arrows?

*In a directed graph, every parent is a direct cause of all its children.*

- By adding causal edge assumption, we have this DAG to represent not only statistical dependencies, but causal relations
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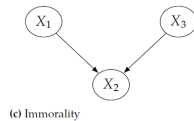
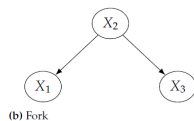
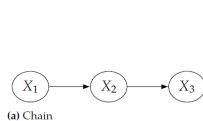
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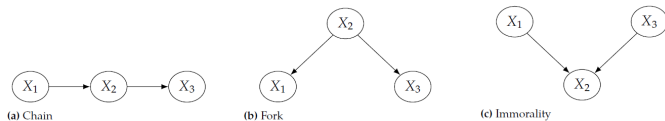
- Now we introduce some building blocks of the causal graph



- Flow of association is symmetric:  $X_1$  and  $X_3$  are associated in both chain and fork (but not immortality)
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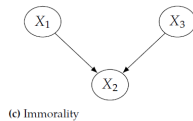
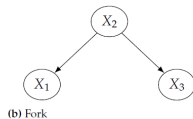
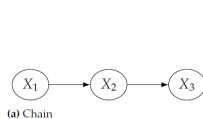
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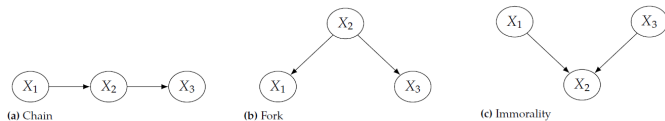
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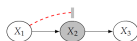


Figure 3.14: Chain with association blocked by conditioning on  $X_2$ .

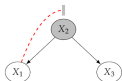


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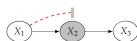


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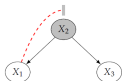


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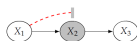


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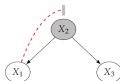


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- We call  $X_2$ , the child of a immorality, as a *collider*

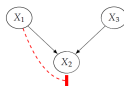


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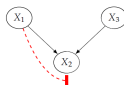


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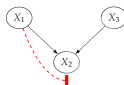


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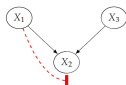


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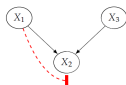


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## Definition (Blocked Path)

A path between  $X$  and  $Y$  is blocked by a conditioning set  $Z$  if either of the following is true:

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- Association flows along unblocked paths, does NOT flow along blocked paths!

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2. There is a collider  $W$  that both itself and its descendants are not conditioned on in  $Z$ ;

- Association flows along unblocked paths, does NOT flow along blocked paths!

## Definition (d-separation)

Two sets of nodes  $X$  and  $Y$  are d-separated by a set of nodes  $Z$  if all of the paths between nodes in  $X$  and nodes in  $Y$  are blocked by  $Z$

- **d-separation means conditional independence!!**
- All association flows between  $X$  and  $Y$  are blocked by  $Z$

# DAG Approach: Blocked Path and d-separation

- Theorem 1.2.4, 1.2.5 in Pearl (2009), Theorem 3.1 in Neal (2020)

If  $X$  and  $Y$  are independent in a DAG  $G$  conditional on  $Z$ , then  $X$  and  $Y$  are independent conditional on  $Z$  in every distribution compatible with  $G$ .

$$X \perp\!\!\!\perp Y \mid Z \text{ in } G \Rightarrow X \perp\!\!\!\perp Y \mid Z, \forall P \text{ compatible with } G$$

Conversely, if  $X$  and  $Y$  are independent conditional on  $Z$  in all  $P$  compatible with  $G$ , then  $X$  and  $Y$  are independent in  $G$  conditional on  $Z$ .

$$\forall P \text{ compatible with } G, X \perp\!\!\!\perp Y \mid Z \Rightarrow X \perp\!\!\!\perp Y \mid Z \text{ in } G$$

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- Associations flow along unblocked paths
- Causations flow along directed unblocked paths
- Identification: how to net causation out of associations?
- By ensuring that **there is no non-causal association between X and Y!**
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# DAG Approach: do-operator

- We define operator " $do(T = t)$ " as an intervention to give the whole population treatment  $t$
- We denote it in terms of potential outcomes as:

$$P(y|do(t)) = P(Y = y|do(T = t)) = P(Y(t) = y) \quad (6)$$

- $P(y|do(t))$  means the distribution of the potential outcome  $Y(t)$
- Identification of a causal model: If we can reduce an expression  $Q$  with  $do$  to one without  $do$ , then  $Q$  is identifiable.
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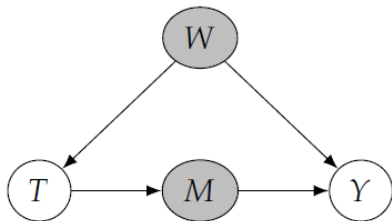
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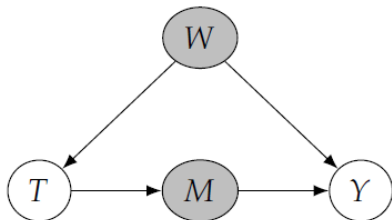
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- Non-directed unblocked paths from  $T$  to  $Y$  are "backdoor paths"
- If some variable set  $W$  blocks all backdoor paths from  $T$  to  $Y$  and does not contain any descendants of  $T$ , we say  $W$  satisfies "the backdoor criterion"



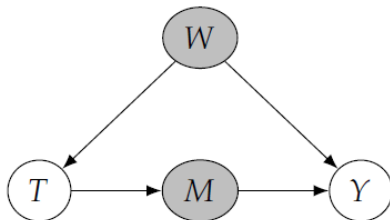
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# DAG Approach: Backdoor Adjustment

- Backdoor Adjustment Theorem

$W$  satis. as the backdoor criterion, we can identify the causal effect of  $X$  on  $Y$  by

$$P(Y|do(X)) = \int_{\mathcal{W}} P(Y|X,W)P(W)$$

- $W$  is what we usually call "control variables"
- The backdoor criterion is similar to the "selection on observables" assumption

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## Theorem (Backdoor Adjustment)

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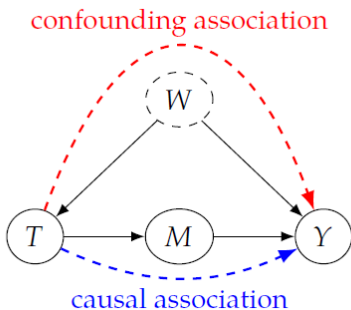
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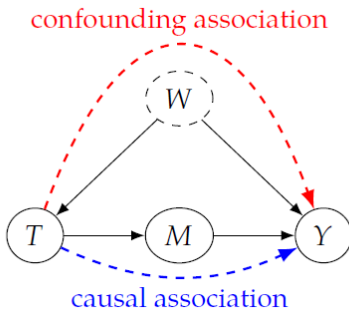
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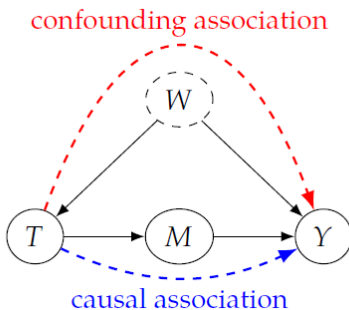
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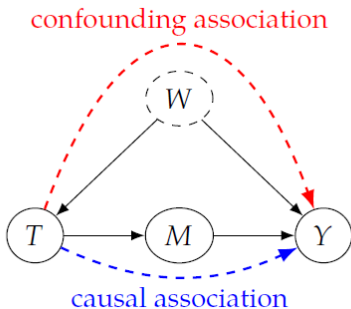
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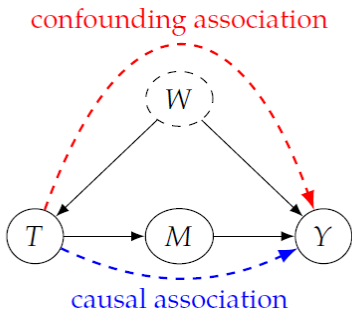
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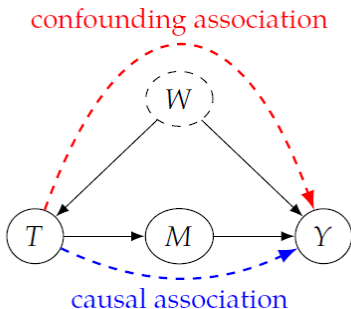
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- If  $W$  is unobserved, we can identify effect of  $T$  on  $Y$  in three steps
  1. Identify effect of  $T$  on  $M$
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  3. Combine step 1 and 2



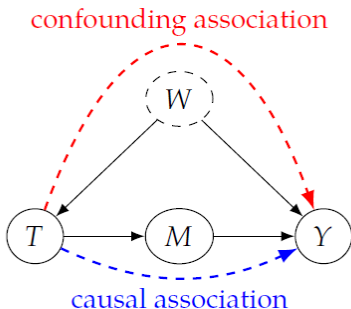
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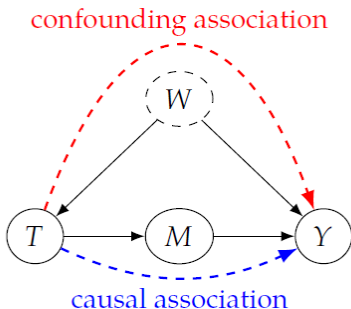
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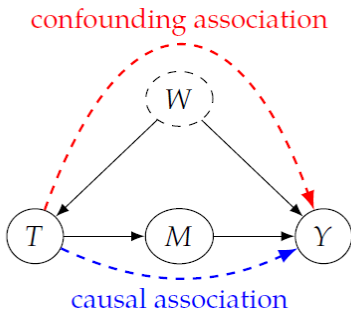
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# DAG Approach: Frontdoor Adjustment

## Definition (Frontdoor Criterion)

A set of variables  $M$  satisfies the frontdoor criterion relative to  $T$  and  $Y$  if:

1.  $M$  completely mediates the causal effect of  $T$  on  $Y$ ;
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## Theorem (Frontdoor Adjustment)

If  $T$ ,  $M$ ,  $Y$  satisfy the frontdoor criterion, then we have

$$P(y|do(t)) = \sum_m P(m|t) \sum_{t'} P(y|m, t') P(t')$$

- We can identify the original treatment effect if we have a complete mediator

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- But backdoor and frontdoor criteria are just sufficient conditions for causal identification
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- Can we find a set of necessary conditions?
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# DAG Approach: Non-parametric Identification

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*A causal effect  $Q$  is identifiable in a model characterized by a graph  $G$  if there exists a finite sequence of transformations, each conforming to one of the inference rules 1, 2, or 3, that reduce  $Q$  into a standard ("do"-free) probability expression involving observed quantities.*

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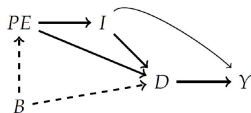
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# DAG Approach: An Example

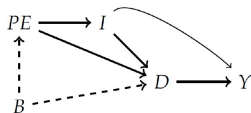
- An example: College ( $D$ ) return on wages ( $Y$ )
- Which variable do we need to control for?



- ▶  $PE$ : parental education
- ▶  $I$ : family income
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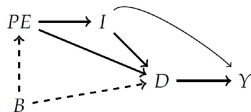
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# DAG in Economics: Clarity

Pro 1: Clarity

- Unconfoundedness

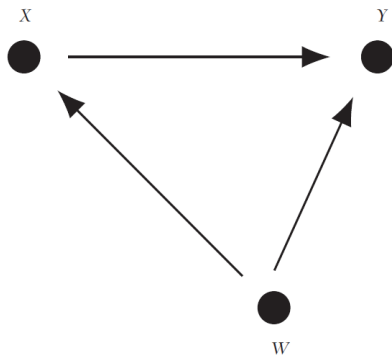


Figure 2. Unconfoundedness

# DAG in Economics: Clarity

- IV strategy

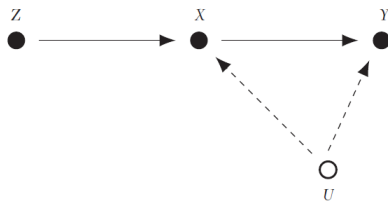


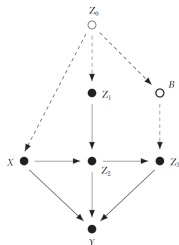
Figure 3. Instrumental Variables

# DAG in Economics: Complicated Model

Pro 2: Tool to analyze complicated causal model

- An example of a complicated model

A: Original



B: Two additional links

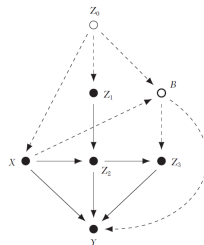


Figure 4. Two Examples of Complex DAGs

# DAG in Economics: Complicated Model

- Structural Equation Modeling
- Given a DAG, we write down a linear equation system

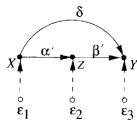


Figure 5.13 Diagram representing model  $M'$  of (5.12)–(5.14).

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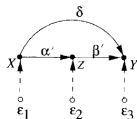


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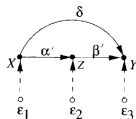


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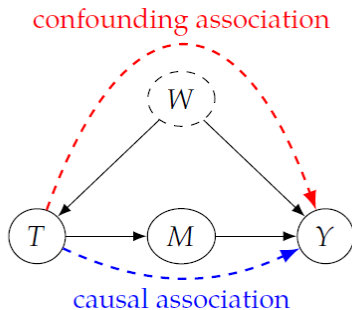
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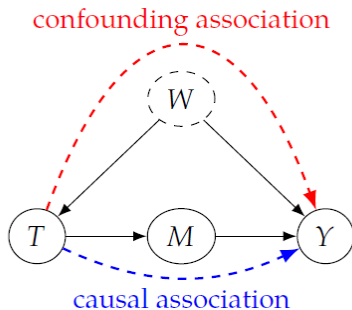
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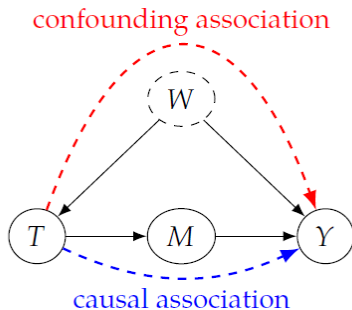
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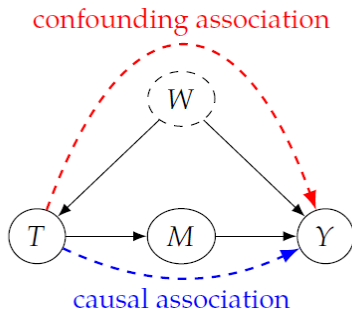
- What if  $T$  affects  $Y$  in other ways?
- What if some unobserved  $U$  affects both  $Y$  and  $M$ ?
- What if  $W$  can also affect  $M$ ?



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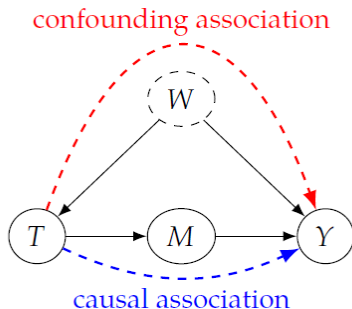
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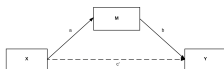
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# DAG in Economics: Mediation

## Pro 4: Systematic analysis of mediation effect

- DAG may shed lights on identifying mediation effect
- The question remains: we need to impose strong causal structure assumption
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- This is a typical behavior of regression monkey
- DAG allows you to "have a causal structure" based on your economic context

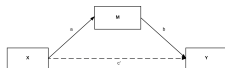




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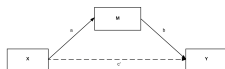
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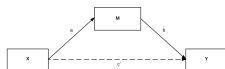
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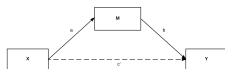
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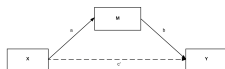
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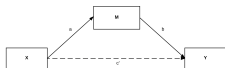
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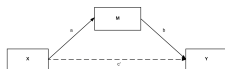
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Con 2: DAG does not fit into IV very well

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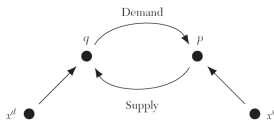


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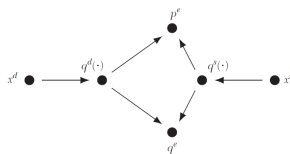
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- DAG by definition is not cyclical
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A: Demand and Supply I



B: Demand and Supply II

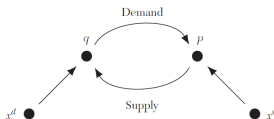


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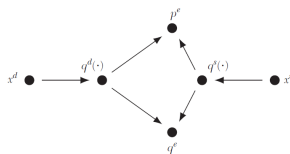
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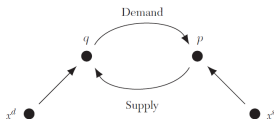


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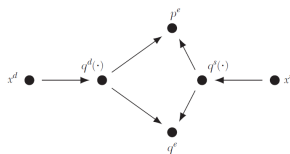
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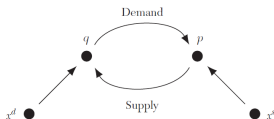


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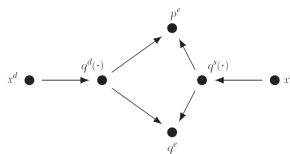
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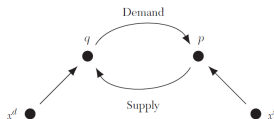


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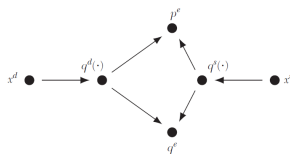
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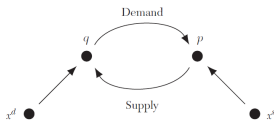


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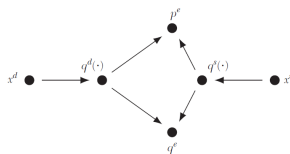
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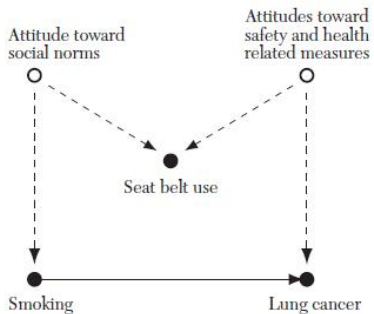
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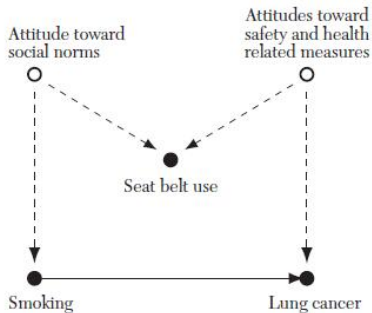
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- This is the only applied ECON paper I've ever read using DAG and Bayesian Networks
- Sadly, in his latest version, Pinto deletes all DAG stuffs...
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# Final Conclusion

- DAG approach fully deserves the attention of all economists
- It has advantages in clearly illustrating causal structures, guiding the selection of controls, and dealing with models with large number of variables
- However, it still has many weaknesses compared with PO in applying to economics
- Especially, it lacks of concrete examples in applying this method in economics
- It is still an open question to all economists! Chances here!

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