Frontier Topics in Empirical Economics: Week 10
Regression Discontinuity Design

Zibin Huang [1]

[1]College of Business, Shanghai University of Finance and Economics

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Introduction

- Assume that we want to examine the education quality of PKU and FDU
- The average wage for PKU graduates is 200,000 RMB/year
- The average wage for FDU graduates is 150,000 RMB/year
- Does this mean that PKU results in higher human capital growth than FDU?

Introduction

- No. Since better students select into PKU
- Self-selection is always a problem in economic research
- Is school A more efficient than school B?
- Or just because they admit students with better initial quality?
- How to deal with this issue?

Introduction

- Of course you can always construct a selection model structurally
- But there is another design-based approach:
  Regression Discontinuity Design (RDD)
- The intuition for RDD is simple
- Draw PKU students just above the PKU admission line and FDU students just below it
- They are students who enroll in PKU/FDU by chance, thus, similar in ability
- Then compare their results

Sharp RD

- Let's first consider a simple case: Sharp RD
- In Sharp RD, treatment rule is deterministic
- That is, you are definitely treated if you surpass the threshold
- Conversely, you are definitely not treated
- There is no uncertainty in treatment assignment

Sharp RD

- Suppose that we have treatment D_i determined by some x_i

$$D_i = \mathbf{1}(x_i \geq x_0) = \begin{cases} 1, & \text{if } x_i \geq x_0 \\ 0, & \text{if } x_i < x_0 \end{cases}$$

- x_i is called running variable
- x_0 is a known threshold or cutoff
- D_i is a deterministic function of x_i

Sharp RD

- We compare samples just above x_0 and just below x_0
- This is a special case of matching
- In conventional matching, we compare samples with identical covariates
- In RD, we compare samples within a small neighborhood at treatment threshold

Sharp RD

- We can write a simple model for this RD

$$Y_i = f_0(x_i)\mathbf{1}(x_i < x_0) + f_1(x_i)\mathbf{1}(x_i \geq x_0) + \rho D_i + \epsilon_i$$

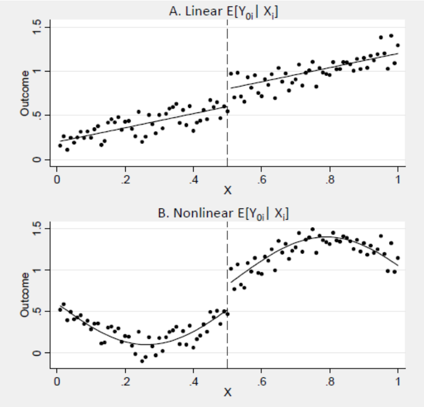
- $f_0(x_i)$ is the smoothing function below the threshold
- $f_1(x_i)$ is the smoothing function above the threshold
- They are used to fit the trend far away from the cutoff
- D_i is the treatment indicator, jumping at $x_i = x_0$

Sharp RD

- We can choose different smoothing functions for f_0 and f_1
- The simplest ones are linear and quadratic functions

Sharp RD

- Here are two examples from Angrist and Pischke (2009), Page 255



Sharp RD

- We can also use non-parametric and semi-parametric functions introduced in Week 2 lecture, which are more flexible
- The most recommended and commonly used one is the Local Linear/Quadratic Regression
- As we have discussed, there is a bias-variance tradeoff
- If you choose complicated smoothing function, you may lose your accuracy
- If you choose too simple smoothing function, you may get bias

Sharp RD

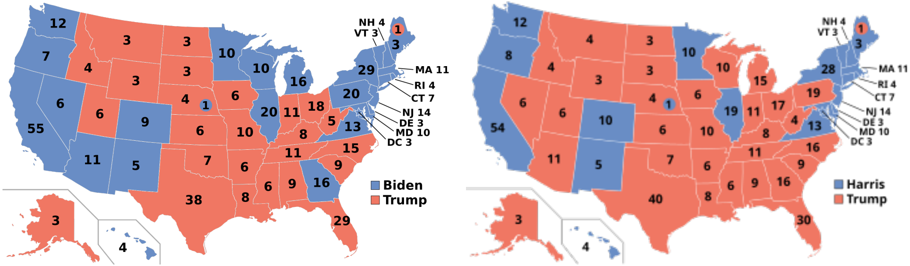
- But remember, effective sample size is usually limited in RD
- You are effectively using a small neighborhood around the cutoff
- So, do not use too complicated smoothing models
- Specifically, Gelman and Imbens (2019) claim that you should avoid using high-order polynomial (over third order)
  - It leads to noisy estimates (Runge's phenomenon)
  - RDD is very sensitive to the degree of the polynomial
  - Coverage of confidence intervals is smaller than nominal

Sharp RD

- An interesting example of Sharp RD is Lee (2008)
- What is the advantage for the party incumbency on reelection?
- Hard to identify since a party may have larger group of supporters for many reasons other than incumbency
- Blue state vs. Red state vs. Swing state

Sharp RD

- Different parties are advantaged in different regions due to ideology, history, religion... reasons



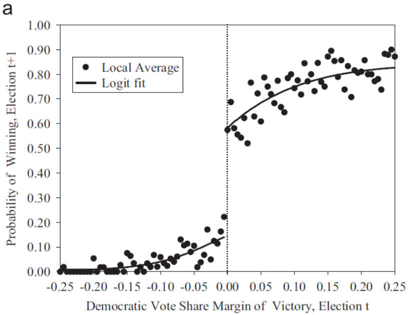
(a) 2020

(b) 2024

Figure: U.S. General Election Map

Sharp RD

- But for elections with very close results, winners and losers are similar
- Lee (2008) considers the probability of Democratic winning in regions where Democratic candidates won by small shares



Fuzzy RD

- A more complicated case is Fuzzy RD
- In Fuzzy RD, treatment assignment is no longer deterministic
- There is uncertainty in being treated or not
- By passing the threshold, you have larger probability to get treated

Fuzzy RD

- Discontinuity in treatment probability, but not treatment

$$P(D_i = 1|x_i) = \begin{cases} g_1(x_i) \text{ if } x_i \geq x_0 \\ g_0(x_i) \text{ if } x_i < x_0 \end{cases}, \text{ where } g_1(x_0) \neq g_0(x_0)$$

- Let's assume that $g_1(x_0) > g_0(x_0)$ WLOG
- Thus, surpassing the cutoff makes treatment more likely

Fuzzy RD

- Denote $T_i = \mathbf{1}(x_i \geq x_0)$ as the indicator of whether passing the cutoff
- Then, we can naturally write Fuzzy RD as a 2SLS
- Treatment D_i is endogenous variable, cutoff indicator T_i is instrument
  - First stage: treatment D_i on cutoff indicator T_i
  - Second stage: outcome variable on first stage fitted value
- The smoothing function f should be included in both stages
- Very simple to implement RD in Stata: Packages such as *rdrobust*
- It helps you to implement bias-corrected CI with optimal bandwidth in Calonico, Cattaneo, and Titiunik (2014)
- You can also try optimal bandwidth in Imbens and Kalyanaraman (2012)

Non-parametric Identification of RD

- We have already introduced how to implement RD method
- And intuitively discussed its identification source
- But what kind of causal effect we are identifying?
- What exactly are its identification assumptions?

Non-parametric Identification of RD

- Let's go to a classic study in RDD, Hahn, Todd, and Van der Klaauw (2001)
- Do not say that you understand RDD if you never read this paper

Non-parametric Identification of RD

- Denote y_{1i}, y_{0i} as the potential outcomes, x_i as the treatment
- We have an outcome $y_i = \alpha_i + x_i \cdot \beta_i$
- Thus, $\alpha_i \equiv y_{0i}, \beta_i \equiv y_{1i} - y_{0i}$
- Assume that we have a running variable z_i
  - In Sharp design, we have $x_i = f(z_i)$ discontinuous at z_0
  - In Fuzzy design, we have $P(x_i = 1|z_i) = f(z_i)$ discontinuous at z_0

Assumption (Fuzzy RD) in Hahn, Todd, and Van der Klaauw (2001)

(i) The limits $x^+ \equiv \lim_{z \to z_0^+} E[x_i|z_i = z]$ and $x^- \equiv \lim_{z \to z_0^-} E[x_i|z_i = z]$ exist; (ii) $x^+ \neq x^-$

Non-parametric Identification of RD

- First, consider the simple case of constant treatment effects
- $\beta_i = \beta$ across individuals
- Assume that mean untreated potential outcome is continuous at the cutoff
- That is, mean of other confounders is continuous at the cutoff

Assumption (A1) in Hahn, Todd, and Van der Klaauw (2001)

$E[\alpha_i | z_i = z]$ is continuous in z at z_0

Non-parametric Identification of RD

- We can prove that β is non-parametrically identified

Theorem 1 in Hahn, Todd, and Van der Klaauw (2001)

Suppose that β_i is fixed at β. Further suppose that Assumptions (RD) and (A1) hold. We then have: $\beta = \frac{y^+ - y^-}{x^+ - x^-}$, where $y^+ \equiv \lim_{z \to z_0^+} E[y_i | z_i = z]$ and $y^- \equiv \lim_{z \to z_0^-} E[y_i | z_i = z]$

- Using an IV-style method, we can pin down the treatment effect

Non-parametric Identification of RD

- Next, we go to more complicated heterogeneous treatment effect case
- We need one more assumption, not only α is continuous at z_0, but also β

Assumption (A2) in Hahn, Todd, and Van der Klaauw (2001)

$E[\beta_i | z_i = z]$ is continuous at $z = z_0$

Non-parametric Identification of RD

- A1 and A2 are different
- A1 is an exogeneity assumption; A2 is a no treatment sorting assumption
- A1 says there is no systematic difference in y_{0i} around the cutoff
- A2 says there is no systematic difference in $y_{1i} - y_{0i}$
- Violation examples:
  - A1: If students with very high ability can control their scores to be just above the cutoff line
  - A2: If students with high return are more likely to select into treatment

Non-parametric Identification of RD

- Then we have the following result

Theorem 2 in Hahn, Todd, and Van der Klaauw (2001)

Suppose that x_i is independent of β_i conditional on z_i near z_0. Further suppose that Assumptions (RD), (A1), and (A2) hold. We then have: $E[\beta_i | z_i = z_0] = \frac{y^+ - y^-}{x^+ - x^-}$

Non-parametric Identification of RD

- Theorem 2 tells us that under heterogeneous TE, if
  - Other confounding factors are continuous at the cutoff (A1)
  - There is no sorting over returns at the cutoff (A2)
- Then we can identify the ATT for individuals around the cutoff
- However, no sorting is a strong assumption under Fuzzy RD
- Individuals of course choose treatment based on how much they can benefit
- Just like Roy model tells us

Non-parametric Identification of RD

- Let's see what will happen if we drop it
- We invoke a set of assumptions similar to Imbens and Angrist (1994) on LATE

Assumption (A3) in Hahn, Todd, and Van der Klaauw (2001)

(i) $(\beta_i, x_i(z))$ is jointly independent of z_i near z_0. (ii) There exists $\epsilon > 0$ such that $x_i(z_0 + e) \geq x_i(z_0 - e)$ for all $0 < e < \epsilon$

- (i) says that given choice x_i, treatment effect β_i is independent of z_i near z_0
- Running variable z can only affect y through changing treatment x
- Test scores only affect wage through changing whether you can be admitted to PKU (exclusion restriction)
- (ii) says that in a small neighborhood around the cutoff, we have monotonicity

Non-parametric Identification of RD

- Under exclusion restriction and monotonicity, we have:

Theorem 3 in Hahn, Todd, and Van der Klaauw (2001)

Suppose that Assumptions (RD), (A1), and (A3) hold. We then have:
$\lim_{e \to 0^+} E[\beta_i | x_i(z_0 + e) - x_i(z_0 - e) = 1] = \frac{y^+ - y^-}{x^+ - x^-}$

- Theorem 3 says that we can identify LATE under a set of assumptions similar to Imbens and Angrist (1994)
- This LATE has two parts to be "Local"
  - Individuals who change their choice around cutoff (Compliers)
  - Individuals around the cutoff

Non-parametric Identification of RD

- From this analysis of identification of RD
- We can derive what conditions we have to validate
- First, we need to check the existence of the discontinuity
- Draw the figure with x-axis as running variable, y-axis as treatment
- Draw the figure with x-axis as running variable, y-axis as outcome
- Visually detect the discontinuity

Non-parametric Identification of RD

- Second, implement balance test for samples just below and just above the cutoff
- Other variables or confounders should be similar or continuous around the cutoff
- Additionally, check the density of samples around the cutoff
- Make sure there is no bunching to either one side of it
- Good students should not control their scores to just a little above the threshold

Application of RD: He, Wang, and Zhang (2020)

- The paper report this week is He, Wang, and Zhang (2020)
- It estimates the effect of environmental regulation on firm productivity in China
- The basic idea is very interesting
- Monitoring stations only capture emissions from upstream regions
- Thus, local gov officials enforce tighter environmental standards on firms just upstream rather than just downstream
- It gives a natural RDD setting

Extension of RDD: RKD

- An interesting extension of RDD is Regression Kink Design (RKD)
- Rather than using the discontinuity on treatment, we employ the kink on treatment
- The jump is no longer on the level, but the slope
- Or we say, the treatment probability derivative has a discontinuity (second order)

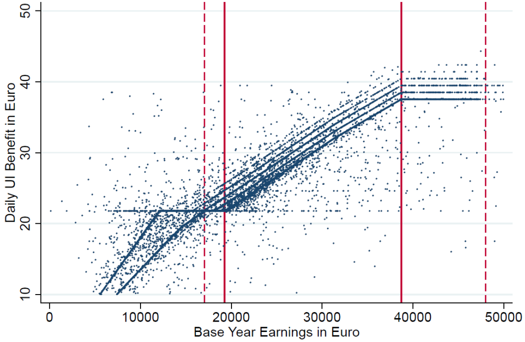
Extension of RDD: RKD

- Consider an example from Card et al. (2015) and Card et al. (2017)
- In many countries, workers can get compensation when they are unemployed
- This is called unemployment benefit (UI)
- The amount of UI depends on the wage of your last job
- If your last wage is too low, there is a minimum benefit level
- There is also a maximum value for UI (Bill Gates will not get billions once he is unemployed)

Extension of RDD: RKD

- Here is a figure for UI distribution in Austria
- Two kinks are noticeable: Minimum and Maximum

Figure 1: UI Benefits in 2004 Austria



Extension of RDD: RKD

- A controversial issue is that too generous UI can incentive workers not to search for new jobs
- It is important to investigate the relation between UI benefit B and unemployment duration Y
- Denote V as the wage of the last job, the running variable; U as an error term
- We have $Y \equiv y(B, V, U)$ as the outcome function
- In a sharp kink design, B is a deterministic function of V: $B = b(V)$ with a slope change at $V = 0$
- Here we normalize the kink to $V = 0$ to simplify the notation

Extension of RDD: RKD

- Assumption 1: (i) U is bounded; (ii) y is continuous and partially differentiable w.r.t. b and v, $y_b(b, v, u)$ is continuous (Regularity)
- Assumption 2: $y_v(b, v, u)$ is continuous around the kink $v = 0$ (Exclusion). The kink exists only for $b(v)$, but not for the effect of v directly on y.
- Assumption 3: Treatment assignment rule $b(v)$ is known, continuous, and has a kink at $v = 0$ (Kink existence)
- Assumption 4: Conditional density $f_{V|U}(v)$ and its partial derivative w.r.t v are continuous around the kink $v = 0$ (No kink for confounders)

Extension of RDD: RKD

- Then we have the non-parametric identification of RKD

Proposition 1 in Card et al. (2015)

In a valid Sharp RKD, that is, when Assumptions 1-4 hold:
(a) $P(U \leq u|V = v)$ is continuously differentiable in v at $v = 0$ $\forall u \in I_U$, where I_U is the neighborhood of the kink.
(b) $E[y_b(b_0, 0, U)|V = 0] = \frac{\lim_{v \to 0^+} \frac{dE[Y|V=v]}{dv}\Big|_{V=V_0} - \lim_{v \to 0^-} \frac{dE[Y|V=v]}{dv}\Big|_{V=V_0}}{\lim_{v \to 0^+} \frac{db(v)}{dv}\Big|_{V=V_0} - \lim_{v \to 0^-} \frac{db(v)}{dv}\Big|_{V=V_0}}$

- Sharp RKD is dividing slope change of $E[Y|V]$ by slope change of $b(v)$
- On the contrary, RDD divides level by level
- Sharp RKD identifies the ATE for individuals with $B = b_0, V = 0$

Extension of RDD: RKD

- The intuition here is as follows
- A change in the slope of treatment probability results in a change in the slope of average outcome
- If there is no change of slope for unobserved confounders
- We can attribute all changes in outcomes to changes of treatment

Extension of RDD: RKD

- What about Fuzzy case?
- The result is very complicated, but with no surprising intuition
- In a Fuzzy RKD, we identify a LATE for individuals who have UI slope changes at the kink
- The larger you change, the larger weight you have
- Of course, we have to invoke some monotonicity assumption

Conclusion

- When you have a discontinuity in treatment, you can use RDD
- Sharp RDD is matching
  - Using samples around the cutoff
  - It identifies ATT for individuals around the cutoff
- Fuzzy RDD is IV
  - Using cutoff indicator as instrument
  - It identifies LATE for compliers around the cutoff
- When you have a discontinuity in treatment slope, you can use RKD
- It also identifies ATT and LATE in Sharp and Fuzzy settings, respectively

Conclusion

- In practice, remember the following tips:
  - Do not use high-order polynomials as smoothing functions
  - A common way is to use local linear regression
  - Using packages in Stata to give you optimal bandwidth and bias-corrected inference
  - Implement balance test both visually and statistically to validate your assumptions

References

Angrist, Joshua D and Jörn-Steffen Pischke. 2009. *Mostly Harmless Econometrics: An Empiricist's Companion*. Princeton University Press.

Calonico, Sebastian, Matias D Cattaneo, and Rocio Titiunik. 2014. "Robust Nonparametric Confidence Intervals for Regression-discontinuity Designs." *Econometrica* 82 (6):2295–2326.

Card, David, David S Lee, Zhuan Pei, and Andrea Weber. 2015. "Inference on Causal Effects in a Generalized Regression Kink Design." *Econometrica* 83 (6):2453–2483.

———. 2017. "Regression Kink Design: Theory and Practice." In *Regression Discontinuity Designs*, vol. 38. Emerald Publishing Limited, 341–382.

Gelman, Andrew and Guido Imbens. 2019. "Why High-order Polynomials Should Not be Used in Regression Discontinuity Designs." *Journal of Business & Economic Statistics* 37 (3):447–456.

Hahn, Jinyong, Petra Todd, and Wilbert Van der Klaauw. 2001. "Identification and Estimation of Treatment Effects with a Regression-discontinuity Design." *Econometrica* 69 (1):201–209.

He, Guojun, Shaoda Wang, and Bing Zhang. 2020. "Watering Down Environmental Regulation in China." *The Quarterly Journal of Economics* 135 (4):2135–2185.

Imbens, Guido and Karthik Kalyanaraman. 2012. "Optimal Bandwidth Choice for the Regression Discontinuity Estimator." *The Review of Economic Studies* 79 (3):933–959.

Imbens, Gw and Jd Angrist. 1994. "Identification and Estimation of Local Average Treatment Effects." *Econometrica* 62 (2):467–475.

Lee, David S. 2008. "Randomized Experiments from Non-random Selection in US House Elections." *Journal of Econometrics* 142 (2):675–697.