Frontier Topics in Empirical Economics: Week 12
Discrete Choice Model II

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Maximum Likelihood Estimation

- We have introduced the Logit model
- Now we consider how to estimate it

$$P_{ni} = \frac{e^{V_{ni}}}{\sum_j e^{V_{nj}}} \tag{1}$$

- There are several ways for the estimation
- A simple and naive way is to log-linearize it:

$$lnP_{ni} = V_{ni} + \underbrace{ln(\sum_j e^{V_{nj}})}_{FE_n} \tag{2}$$

- When V_{ni} is linear, we run an OLS with fixed effect at n level

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Maximum Likelihood Estimation

- Widely used in trade literature, especially in estimating the gravity equation
- Very easy to implement
- However, this is NOT the main method people usually use, especially in labor:
  - In data, when $P_{ni} = 0$, log values are undefined
    Granular setting issues in spatial economics (Dingel and Tintelnot, 2020)
  - Absorbed by the FE: any parameters for n level variables in V_{ni} cannot be estimated
  - The OLS only uses part of the information, not efficient

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Maximum Likelihood Estimation

- The best method is MLE

$$\text{Likelihood Function: } L(\beta) = \prod_{n}^{N} \prod_{i} (P_{ni})^{y_{ni}}$$

$$\text{Log Likelihood Function: } LL(\beta) = \sum_{n=1}^{N} \sum_{i} y_{ni} ln P_{ni}$$

$$\text{MLE Estimator: } \hat{\beta}_{MLE} = argmax_{\beta} LL(\beta)$$

- y_{ni} is whether choice i is chosen in the data by individual n
- We choose β to maximize the probability of observing such data of y_{ni}

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- We can have a closed-form optimal solution
- But very often, there is no closed-form solution in MLE
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Maximum Likelihood Estimation

- The idea of numerical non-linear optimization is: Guess → Update → Iterate
  - Step 1: Make an initial guess of β^0
  - Step 2: Use some updating rule to update β^0 to β^1, search for the optimal point
  - Step 3: Keep update in step 2 ($\beta^2, \beta^3, \ldots, \beta^k$) until we find the solution
- Then the question is, how to search and update?
- Today we will give a very brief introduction to numerical methods used in Economics

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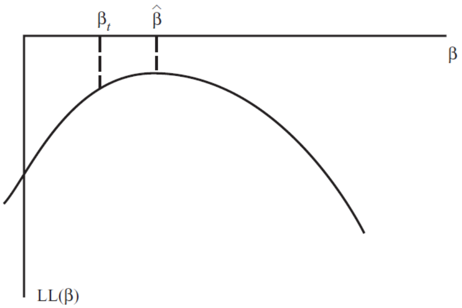

Figure 8.1. Maximum likelihood estimate.

How to search? Follow the derivative!

Maximum Likelihood Estimation

- Let's define $K \times 1$ gradient for each iteration β^t as:

$$g^t = \left(\frac{\partial LL(\beta)}{\partial \beta}\right) \tag{3}$$

- This is the vector of first order derivatives for an vector β
- Define $K \times K$ Hessian matrix for β^t as:

$$H^t = \left(\frac{\partial g^t}{\partial \beta'}\right) = \left(\frac{\partial^2 LL(\beta)}{\partial \beta \partial \beta'}\right) \tag{4}$$

- This is the matrix of second order derivatives (including cross derivatives) for β

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Maximum Likelihood Estimation

- For all gradient-based optimization method, we have the same form of updating rule:

$$\beta^{t+1} = \beta^{t} + \lambda M g^{t} \quad (5)$$

- g^{t} is the gradient, controlling the updating direction
- λ is a scalar called step size, M is a $K \times K$ matrix
- They control the speed of updating

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Maximum Likelihood Estimation

- The most famous method is Newton-Raphson (NR):

$$\beta^{t+1} = \beta^{t} + \lambda(-H^{t})^{-1} g^{t} \tag{6}$$

- The NR method uses Hessian $(-H^{t})^{-1}$ as the speed matrix
- This is actually very intuitive
  - Gradient tells us the direction:
    Positive $\Rightarrow \beta^{t+1} \uparrow$; Negative $\Rightarrow$ need to decrease $\beta^{t+1} \downarrow$
  - Hessian is the curvature of the function:
    More curved means the slope changes quickly: need to be more conservative
- λ can help us adjust step size in case it is too large (update past maximum)

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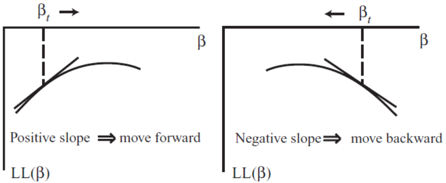

Figure 8.2. Direction of step follows the slope.

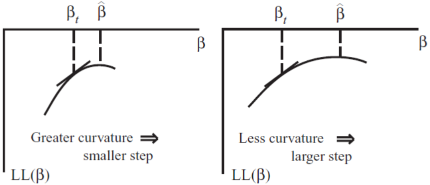

Figure 8.3. Step size is inversely related to curvature.

Maximum Likelihood Estimation

- There are two drawbacks of the NR method
  - Calculation of Hessian is computationally intensive
  - No guarantee to follow the direction of gradient: If LL is not globally concave, Hessian is not necessarily positive definite!
- There are other methods using different speed matrix to overcome these issues
- For more details, please refer to Chapter 8 in Train (2009)
- When doing MLE, usually we can use packages like *Optim* in Julia, or *fminsearch* in Matlab
- No need to do it by yourself

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Endogeneity in DCM: Issues

- So far we assume the unobserved errors ϵ is independent of the explanatory variables
- But this cannot always be the case
- Assume a case that consumers want to buy cars (BLP, 1995)
- The effect of the car price on the purchase probability is hard to be identified
- Since there can be unobserved car traits correlated with prices (smoothness of ride, brand reputation...)
- These are endogeneity issues in DCM
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- There are M markets with J_m options (brands) in each market
- Utility for consumer n in market j to choose brand m is:

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- p_{jm} price; s_n personal attributes; x_{jm} product attributes; ξ_{jm} unobserved product attributes; ϵ_{njm} i.i.d. T1EV shock
- In simple case, V is linear
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- The idea of BLP employs a two-step approach
- First, add in a product-market level FE, absorb ξ_{jm}
- Estimate the equation with fixed effect
- Second, open the box of product-market level FE, estimate the remaining parameters
- The key point here is that endogeneity happens only at product-market level!

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- We can decompose the observed utility value into

$$V_{njm} = \underbrace{\bar{V}(p_{jm}, x_{jm}, \bar{\beta})}_{\text{varies only over product-market}} + \underbrace{\tilde{V}(p_{jm}, x_{jm}, s_n, \tilde{\beta}_n)}_{\text{varies also over consumer}}$$

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- We define product-market level FE as:

$$\delta_{jm} = \bar{V}(p_{jm}, x_{jm}, \bar{\beta}) + \xi_{jm} \quad (7)$$

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- Equation (8) does not entail any endogeneity
- Step 1: We run a Logit model with jm level FE to estimate parameters $\tilde{\beta}$
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The essence of BLP

- We cannot run IV regression directly in DCM
- We first pack all terms at the level where endogeneity happens into FE
- Then we estimate a DCM with these FEs
- We have estimated FEs, then unpack it and run linear IV regression
- Transform non-linear IV to be linear IV
- BLP tells you how to use an IV in a DCM, but only in a specific model structure.

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- It is a contraction mapping method to numerically estimate the FEs
- This is also a standard method for solving FE with high dimensions
- This procedure rests on that FEs determine predicted market shares for each product in each market
- Therefore, in each iteration, we can set the predicted shares from model to equal actual shares in data

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- Let S_{jm} be the real market share of product j in market m in data (Share of BYD in Shanghai)
- Similarly, define $\hat{S}_{jm} = \sum_n \hat{P}_{njm} / N_m$ as the predicted share from your model
  - $\sum_n \hat{P}_{njm}$ is the predicted total sales of product j in market m
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- Denote δ as the vector of δ_{jm} for all j, m

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- Let's see the Berry Contraction algorithm for Step 1
  - (1) Take an initial guess for parameters $\tilde{\beta}_n$ in $\tilde{V}(p_{jm}, x_{jm}, s_{nm}; \tilde{\beta}_n)$
  - (2) Take an initial guess of δ
  - (3) For each guess of $\tilde{V}^t$ and δ^t, we calculate the choice value $\hat{U}^t$ for each consumer of each product in each market: $\hat{U}^t_{njm} = \delta^t_{jm} + \tilde{V}^t(p_{jm}, x_{jm}, s_{nm}; \tilde{\beta}_n)$
  - (4) Given the calculated choice values, we calculate the predicted choice probability $\hat{P}^t_{njm} = \frac{\exp(\hat{U}^t_{njm})}{\sum_{j'} \exp(\hat{U}^t_{nj'm})}$; $\hat{S}^t_{jm} = \sum_n \hat{P}^t_{njm} / N_m$
  - (5) Given the choice probability, update FEs as follows: $\delta^{t+1}_{jm} = \delta^t_{jm} + ln\left(\frac{S_{jm}}{\hat{S}^t_{jm}(\delta^t)}\right)$
  - (6) Iterate (3)-(5) until convergence of FEs for each guess of $\tilde{\beta}_n$
  - (7) Using traditional non-linear optimization method to iterate (1)-(6) until optimal point is found for $\tilde{\beta}_n$

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- The idea is to separate the estimation of FEs
- In the outer loop (1)-(6), we estimate $\tilde{\beta}_n$ in $\tilde{V}(p_{jm}, x_{jm}, s_m, \tilde{\beta}_n)$ using traditional MLE algorithm
- In the inner loop (3)-(5), for each value of $\tilde{\beta}_n$, we iterate FEs δ_{jm}
- This procedure rests on that FEs determine predicted market shares for each product in each market
- Therefore, in each iteration, we can set the predicted shares from model to equal actual shares in data

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- This algorithm in BLP is a contraction mapping

Definition: Contraction Mapping

Let (X, d) be a metric space. Then a map $T : X \to X$ is called a contraction mapping on X if there exists $q \in [0, 1)$ such that:

$$d(T(x), T(y)) \leq q d(x, y) \;\; \forall x, y \in X$$

- The contraction mapping means a function that squeezes points closer together in a space

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Endogeneity in DCM: 1. BLP

- Then we have the famous Banach fixed-point theorem

Banach fixed-point theorem

Let (X, d) be a non-empty complete metric space with a contraction mapping $T : X \to X$. Then T admits a unique fixed-point x^* in X, that is, $T(x^*) = x^*$. Furthermore, x^* can be found as follows: start with an arbitrary element $x_0 \in X$ and define a sequence $(x_n)_{n \in N}$ by $x_n = T(x_{n-1})$ for $n \geq 1$. Then $\lim_{n \to \infty} x_n = x^*$.

- The existence of a contraction mapping $T \Rightarrow$ Unique fixed point $T(x^*) = x^*$
- We can find x^* by iterate some arbitrary initial x_0 with T
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- What does it mean in this BLP algorithm?
- It means that, as long as the updating iteration is a contraction mapping
- We will converge to the same point of δ, no matter what is our initial guess δ^0
- This terminal point is the fixed point, $\delta^* = F(\delta^*)$
- With δ^*, the predicted share equals the data share

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- This idea is widely used in the computation problems
- For instance, calculating the equilibrium in a complicated model
- With a contraction mapping, we find the fixed-point which pins down the equilibrium
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- BLP is not always feasible (error structure...)
- The algorithm of estimating BLP is complicated (involving some contraction)
- Highly recommend you to read BLP part in Train's book (or better, BLP 1993)
- The second important non-linear IV approach is Control Function (CF)

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- The utility of consumer n buying product j is:

$$U_{nj} = V(x_{nj}, w_{nj}, \beta_n) + \epsilon_{nj}$$

- x_{nj} is endogenous, $x_{nj} \not\perp \epsilon_{nj}$
- We assume that there is an instrument z_{nj}, related with x_{nj} by first stage:

$$x_{nj} = W(z_{nj}, \gamma) + \mu_{nj} \tag{9}$$

- Assume that $\epsilon_{nj}, \mu_{nj} \perp z_{nj}$, $\epsilon_{nj} \not\perp \mu_{nj}$
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- x_{nj} is endogenous, $x_{nj} \not\perp\!\!\!\perp \epsilon_{nj}$
- We assume that there is an instrument z_{nj}, related with x_{nj} by first stage:

$$x_{nj} = W(z_{nj}, \gamma) + \mu_{nj} \tag{9}$$

- Assume that $\epsilon_{nj}, \mu_{nj} \perp\!\!\!\perp z_{nj}$, $\epsilon_{nj} \not\perp\!\!\!\perp \mu_{nj}$
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- We can do a CEF decomposition (given μ_{nj}) for ϵ_{nj}:

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- By construction: $\tilde{\epsilon}_{nj} \perp\!\!\!\perp \mu_{nj}$
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- We call $CF(\mu_{nj}, \lambda)$ a control function, where λ is some parameter
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Endogeneity in DCM: 2. Control Function

- Then we have the utility function as

$$U_{nj} = V(x_{nj}, w_{nj}, \beta_n) + CF(\mu_{nj}, \lambda) + \tilde{\varepsilon}_{nj} \tag{10}$$

- Step 1: Estimate first stage equation (9), get residual of the first stage $\hat{\mu}$
- Step 2: Plug $\hat{\mu}$ in the CF (10), estimate equation (10) using simple Logit
- In step 2, we need to assume a functional form for CF
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Endogeneity in DCM: 2. Control Function

- The logic of CF approach is as follows:
  - We know that instrument z is not correlated with the error ε
  - Thus, endogenous variable x correlates with ε only through first stage error μ
  - Then by controlling the correlated parts of μ and ε, we can eliminate the correlation of x and ε
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Endogeneity in DCM: 3. IV-Probit

- The last method we illustrate is IV-Probit
- It has very strong model structure assumptions
- Consider the following model:

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\begin{aligned}
y_1^* &= \delta_1 z_1 + \alpha_1 y_2 + u_1 \\
y_2 &= \delta_{21} z_1 + \delta_{22} z_2 + v_2 \\
y_1 &= \mathbf{1}(y_1^* > 0)
\end{aligned}
$$

- y_1^* is the latent utility; y_2 is the endogenous variable; z_1 is exogenous control
- (u_1, v_2) is bivariate normal; z_2 is the instrument with $(u_1, v_2) \perp z_2$

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Endogeneity in DCM: 3. IV-Probit

- In IV probit, we employ the assumption of the joint distribution of error (u_1, v_2)
- (u_1, v_2) is bivariate normal, then we can explicitly write down the likelihood function

$$f(y_1, y_2|z) = f(y_1|y_2, z)f(y_2|z) \quad (11)$$

$$= \Phi[\frac{\delta_1 z_1 + \alpha_1 y_2 + (\rho_1/\tau_2)(y_2 - z\delta_2)}{(1-\rho_1^2)^{\frac{1}{2}}}] \quad (12)$$

- We can use MLE to jointly estimate all these parameters
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Endogeneity in DCM: Main Takeaways

Main Takeaways about IV in DCM

- Don't naively use IV method in linear model to solve endogeneity issue in non-linear model! (e.g., 2SLS)
- You can use BLP, CF, or IV-Probit
- BLP fits Logit model, but needs the endogeneity happens at higher level Product-market level in consumers' problem
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