Frontier Topics in Empirical Economics: Chapter 4 Directed Acyclic Graphs

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Introduction

- Causal inference is the central topic of applied economics
- We almost solely focus on potential outcome framework in Economics
- This framework is proposed by Donald Rubin (Imbens and Rubin, 2015; Rubin, 1974) and sometimes called "Rubin Causal Model"

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- Graphical Model is another important method (Pearl, 2009)
- This is a method highly related to computer science and AI

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- How it can be applied to economic research is still a very very open question
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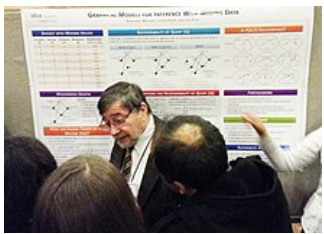

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- Introduce the graphical model and the DAG framework
- Discuss the possible usage of DAG for economists: Pros and Cons
- Compare DAG and PO framework: why PO is still more popular
- An example of using DAG: Pinto (2015)
- Conclusion: How can DAG help applied economics research (open question)

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DAG Approach: Graph

- Graph is a collection of *nodes* and *edges* that connect the nodes.
- Two nodes are called *adjacent* if they are connected by an edge.
- A directed graph's edges go out of a *parent* into a *child*.
- A *path* is any sequence of adjacent nodes, regardless of the direction of the edges. A *directed path* is a path that consists of directed edges that are all directed in the same direction.

(a) Undirected Graph

(b) Directed Graph

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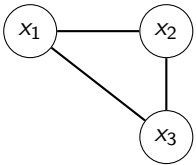

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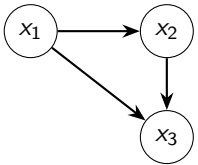

(b) Directed Graph

DAG Approach: Graph

- If there is a directed path that starts at node X and ends at node Y, then X is an ancestor of Y, and Y is a descendant of X.
- If there is no cycle in a directed graph, the graph is called a *directed acyclic graph* (DAG)

(a) No Cycle (DAG)

(b) Cycle (Not a DAG)

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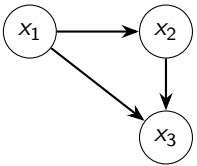

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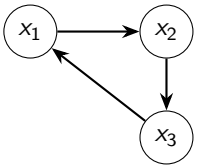

(b) Cycle (Not a DAG)

DAG Approach: Bayesian Networks

- How to connect graphs to causal inference?
- The first step is to connect graphs to statistical relations: Bayesian Networks
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DAG Approach: Bayesian Networks

- For any joint distribution, a Bayesian factorization can be expressed as:

$$P(x_1, x_2, ..., x_n) = P(x_1) \prod_{i \neq 1} P(x_i | x_{i-1}, ..., x_1) \quad (1)$$

- Example: $P(x_1, x_2, x_3) = P(x_1)P(x_2|x_1)P(x_3|x_2, x_1)$
- This is like a chain generated from the Bayesian rule
- We can simplify the model if we assume some dependency structure, e.g. $P(x_3|x_2, x_1) = P(x_3|x_2)$ if $x_1 \perp x_3|x_2$
- When we make more assumptions, we simplify it more

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- Bayesian factorization can be applied to any joint distribution of $(x_1, x_2, \ldots, x_n)$
- With the set of the dependency assumptions, we are giving the joint distribution a structure
- We can use a graph to represent this assumed dependency structure, system of probabilistic relations
- A one-to-one mapping between graph G and probabilistic relations P

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Assumption (Minimality Assumption)

1. Given its parents in the DAG, a node X is independent of all its non-descendants (Local Markov Assumption);
2. Adjacent nodes in the DAG are dependent (Minimal independence).

Theorem (Bayesian Network Factorization)

Given a probability distribution P and a DAG G satisfying the Local Markov Assumption, P factorizes according to G by

$$P(x_1, x_2, ..., x_n) = \prod_i P(x_i | pa_i)$$

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- Local Markov means that the dependence structure is "local" and "Markov"
- Minimal independence means that there is no more independence outside the network shown in the graph
- Bayesian Factorization means that: If P has a statistical structure as shown in G
- We call "G represents P", "G and P are compatible", "P is Markov relative to G"

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- Let's see a simple example
- Assume that we have four variables x_1, x_2, x_3, x_4
- A full decomposition is:

$$P(x_1, x_2, x_3, x_4) = P(x_1)P(x_2|x_1)P(x_3|x_2, x_1)P(x_4|x_3, x_2, x_1) \quad (2)$$

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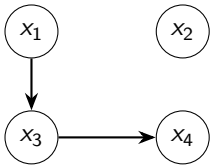

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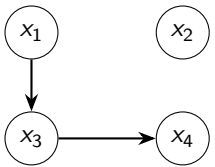

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- Up until now, we consider only statistical dependencies
- What about those arrows?

Assumption (Causal Edges Assumption)

In a directed graph, every parent is a direct cause of all its children.

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DAG Approach: Graphical Building Blocks

- Now we introduce some building blocks of the causal graph

(a) Chain

(b) Fork

(c) Immorality

- Flow of association is symmetric: x_1 and x_3 are associated in both chain and fork (but not immorality)
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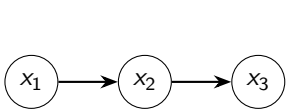

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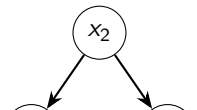

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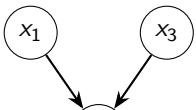

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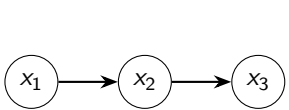

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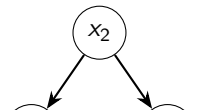

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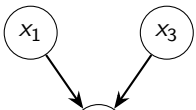

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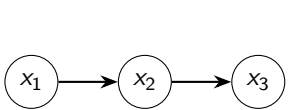

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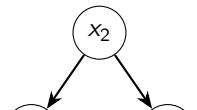

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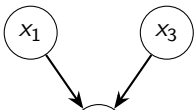

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- We can show that with this graph:

$$P(x_1, x_3|x_2) = P(x_1|x_2)P(x_3|x_2) \tag{4}$$

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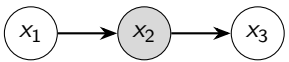

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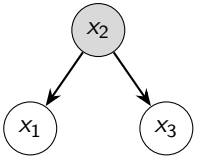

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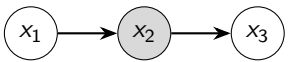

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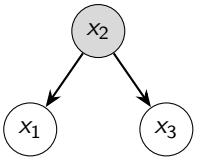

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Proof:

- Bayesian factorization+Local Markov gives us:

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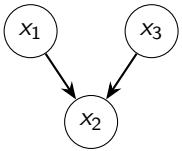

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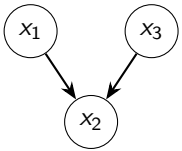

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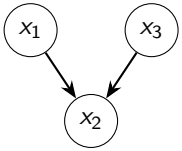

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- Controlling for post-determined variables!
- A simple example: x_1 is good-looking, x_3 is kindness, x_2 is marital status
- Conditioning on $x_2 = 0$ (looking only at unmarried people), you will see a negative relation between x_1 and x_3!
- Well-known as bad control problem in econometrics

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DAG Approach: Blocked Path and d-separation

Definition (Blocked Path)

A path between X and Y is blocked by a conditioning set Z if either of the following is true:
1. *Along the path, there is a chain $\rightarrow W \rightarrow$ or a fork $\leftarrow W \rightarrow$ where $W \in Z$;*
2. *There is a collider W on the path such that neither W itself nor any of its descendants is in Z;*

- Association flows along unblocked paths, does NOT flow along blocked paths!

- d-separation means conditional independence!!
- All association flows between X and Y are blocked by Z

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$$X \perp_G Y | Z \Rightarrow X \perp_P Y | Z, \forall P \text{ compatible with } G$$

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- Causations flow along directed unblocked paths
- Identification: how to net causation out of associations?
- By ensuring that there is no non-causal association between T and Y.
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DAG Approach: do-operator

- We define operator "$do(T = t)$" as an intervention to give the whole population treatment t
- We can denote it in terms of potential outcomes as:

$$P(y|do(t)) = P(Y = y|do(T = t)) = P(Y(t) = y) \tag{6}$$

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- If some variable set W blocks all backdoor paths from T to Y and does not contain any descendants of T, we say W satisfies "the backdoor criterion"

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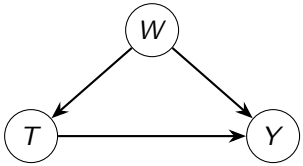

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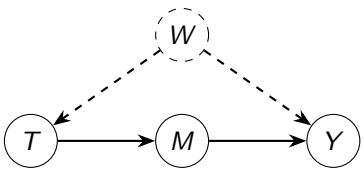

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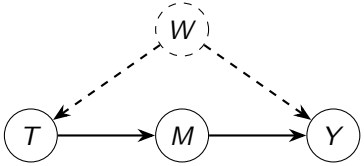

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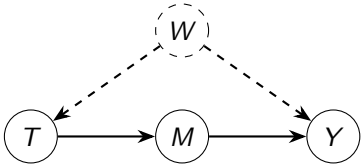

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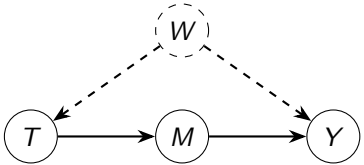

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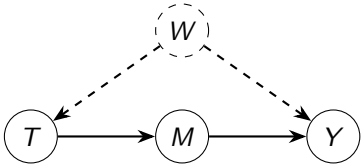

DAG Approach: Frontdoor Adjustment

Definition (Frontdoor Criterion)

A set of variables M satisfies the frontdoor criterion relative to T and Y if:

1. *M completely mediates the causal effect of T on Y;*
2. *There is no unblocked backdoor path from T to M;*
3. *All backdoor paths from M to Y are blocked by T.*

Theorem (Frontdoor Adjustment)

If T, M, Y satisfy the frontdoor criterion, then we have

$$P(y|do(t)) = \sum_m P(m|t) \sum_{t'} P(y|m,t')P(t')$$

- We can identify the original treatment effect if we have a complete mediator

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- But backdoor and frontdoor criteria are just sufficient conditions for causal identification
- They are not necessary
- Can we find a set of necessary and sufficient conditions (a complete characterization)?
- If there is such a set, we can decide whether a causal effect is identifiable or not in any causal system
- Here it comes: do-calculus

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- Denote $G_{\overline{X}}$ as take graph G and then remove all incoming edges to X
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Theorem (Identification of Causal Effect)

A causal effect Q is identifiable in a model characterized by a graph G if and only if there exists a finite sequence of transformations, each conforming to one of the inference rules 1, 2, or 3, that reduce Q into a standard ("do"-free) probability expression involving observed quantities.

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- Rule 1 (deletion of var): $P(y|do(t), z, w) = P(y|do(t), w)$, if $Y \perp_{G_{\overline{T}}} Z|T, W$
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- An example: College (D) return on wages (Y)
- Which variable do we need to control for?

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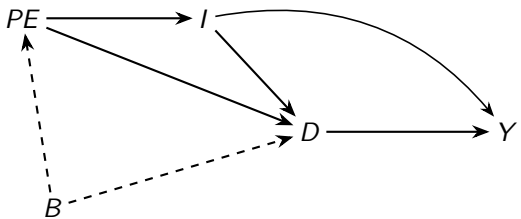

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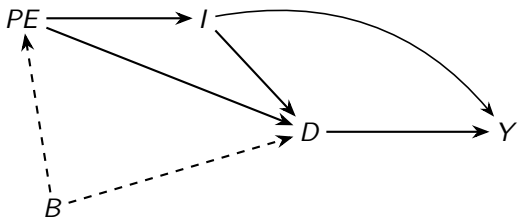

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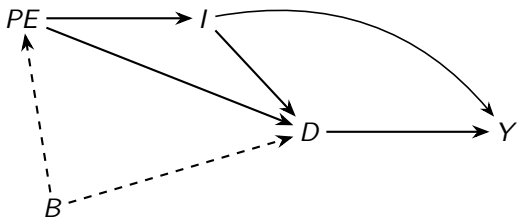

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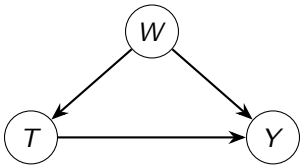
DAG in Economics: Clarity

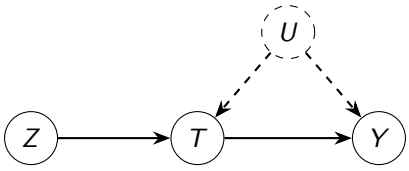
Pro 1: Clarity

- Unconfoundedness



DAG in Economics: Clarity

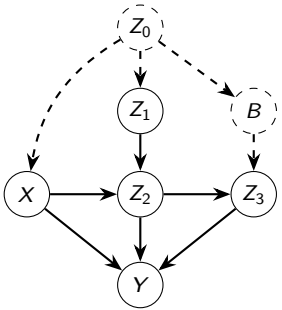
- IV strategy



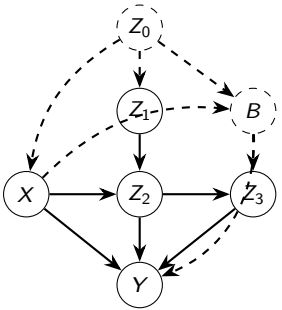
DAG in Economics: Complicated Model

Pro 2: Tool to analyze complicated causal model

- An example of a complicated model (Imbens, 2020)



(a) Original model



(b) Two additional links

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- Given a DAG, we write down a linear equation system (Figure 5.13 in Pearl, 2009)

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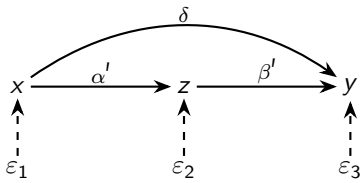

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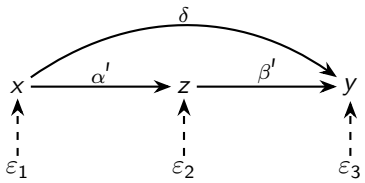

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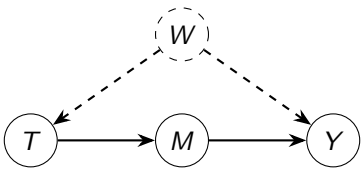

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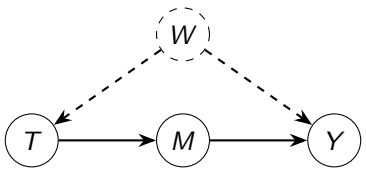

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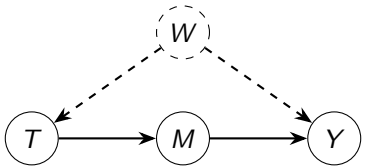

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Pro 4: Systematic analysis of mediation effect

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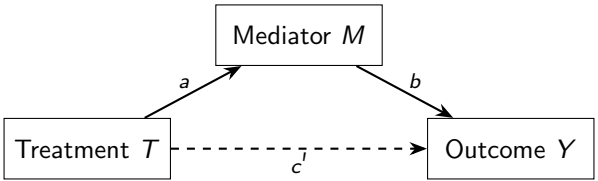

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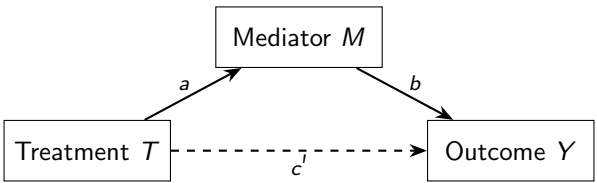

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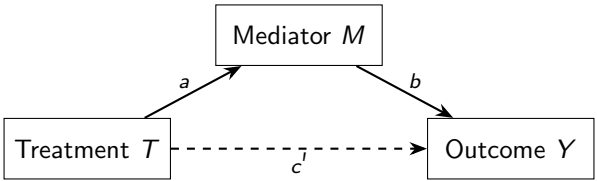

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- As a method, I would say it is out of date, but still a simple/straightforward one
- The problem is that people are using it without thinking!!
- The underlying assumptions are super strong

- If you admit this structure, you can directly identify the effect of T on Y using OLS!
- Why bother to have all those research designs?

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Con 3: DAG cannot capture some equilibrium concept in Economics

- DAG by definition is not cyclical
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- Here is an attempt from Imbens, though not so successful

(a) Demand and supply

(b) Equilibrium representation

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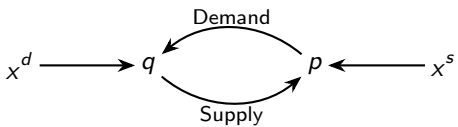

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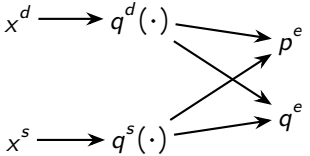

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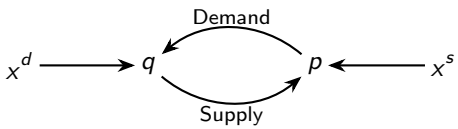

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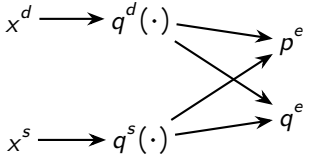

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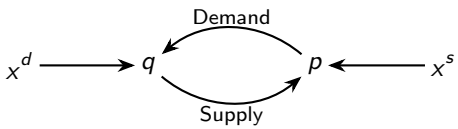

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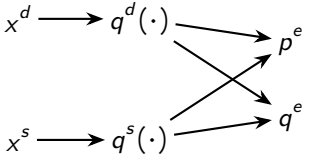

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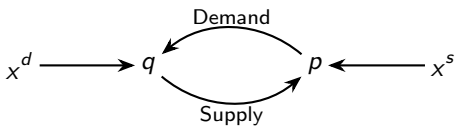

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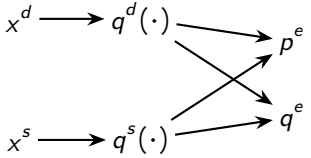

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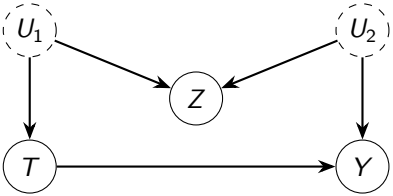

Personal View: Clarity in Illustration

- We can see here the essence is that we should not control for colliders
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- It forces us to firmly and explicitly consider our causal structure and show them in a transparent way

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- How should we select control variables? There are two broad perspectives
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- For causal questions, identification comes first: select controls from the causal structure, then use model fit considerations to refine the specification
- Again a tension between two philosophical approaches about how to understand the world

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