Frontier Topics in Empirical Economics: Chapter 8 Causal Inference with Panel Data I

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Fixed Effect: Panel Data

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- What if we have one more dimension: Time?
- We call it Panel Data
- We can exploit variations across time for the same individual (unit)

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Fixed Effect: FE Settings

- What is the impact of military service on wages?
  Framework follows Angrist and Pischke (2009), Chapter 5
- Person i, time t, wage Y_{it}, military service status D_{it}, ability A_i, covariates X_{it}
- Assume constant TE and linear CEF, we have:

$$Y_{it} = \alpha + \rho D_{it} + A_i'\gamma + X_{it}'\beta + \epsilon_{it} \quad (1)$$

$$E[Y_{it}|A_i, X_{it}, D_{it}] = \alpha + \rho D_{it} + A_i'\gamma + X_{it}'\beta \quad (2)$$

- A_i is the unobserved confounding factor, $\epsilon_{it} \perp\!\!\!\perp D_{it}|A_i, X_{it}$
- How to estimate ρ? Three simple ways

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Fixed Effect: FE Estimator

Method 1: Fixed Effect Estimator

- FE Estimator is a deviation-from-mean estimator
- Step 1: Take individual-level means of both sides of the regression

$$\bar{Y}_i = \alpha + \rho \bar{D}_i + A_i' \gamma + \bar{X}_i' \beta + \bar{\epsilon}_i$$

- Step 2: Subtract the mean from the original regression

$$Y_{it} - \bar{Y}_i = \alpha - \alpha + \rho(D_{it} - \bar{D}_i) + A_i' \gamma - A_i' \gamma + (X_{it}' - \bar{X}_i')\beta + (\epsilon_{it} - \bar{\epsilon}_i) \quad (3)$$

$$= \rho(D_{it} - \bar{D}_i) + (X_{it}' - \bar{X}_i')\beta + (\epsilon_{it} - \bar{\epsilon}_i) \quad (4)$$

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Method 2: Dummy Estimator

- We can add a set of individual dummies
- Saturate across the individual dimension

$$Y_{it} = (\alpha + A_i'\gamma) + \rho D_{it} + X_{it}'\beta + \epsilon_{it}$$

$$Y_{it} = \alpha_i + \rho D_{it} + X_{it}'\beta + \epsilon_{it} \quad (5)$$

- Unobserved A_i is absorbed in dummy α_i
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Fixed Effect: FD Estimator

Method 3: FD Estimator

- We can run the regression using differencing (across time) variables
- Assume that $\Delta Y_{it} = Y_{it} - Y_{it-1}$ means time difference
- Subtracting regression in t by $t-1$, we have:

$$\Delta Y_{it} = \rho \Delta D_{it} + \Delta X_{it}' \beta + \Delta \epsilon_{it} \tag{6}$$

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Fixed Effect: FE, Dummy, and FD Estimator

- All of them employ the variations across time for the same person
- FE and Dummy estimators are identical
  Their point estimations, std errs, and other main statistics are the same
- So we usually call FE and Dummy estimators "FE Model"

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- FE and FD are same in two-period case
- FE and FD are different when $T > 2$
- When ϵ_{it} are serially uncorrelated shocks, FE is more efficient than FD
- Since FD will create serial correlation in the errors: $\Delta\epsilon_{it} = \epsilon_{it} - \epsilon_{it-1}$
  It invalidates the usual inference based on the i.i.d. assumption and damages efficiency
- But when ϵ_{it} follows a random walk, FD is better since the differenced errors are now uncorrelated
  Random walk: $\epsilon_{it} = \epsilon_{it-1} + \psi_{t-1}$; $\psi_{i,t-1}$ is i.i.d.

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DID: TWFE

- For panel data, usually we can control for both individual and time FE

$$Y_{it} = \rho D_{it} + X_{it}'\beta + \lambda_t + \alpha_i + \epsilon_{it} \tag{7}$$

- This is called Two-way Fixed Effect Model (TWFE)
- Difference-in-Differences (DID) is a special case of TWFE model
- In DID, usually some policy is implemented at higher level (Province, City...)
- D_{it} is binary (whether individual i at time t is treated by the policy)
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DID: Settings

- Example: Card and Krueger (1994) Effects of minimum wage on employment
- On April 1, 1992, New Jersey raised the state minimum wage
- But in its neighboring state of Pennsylvania, nothing happened
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$$E[Y_{0ist}|s,t] = \gamma_s + \lambda_t \tag{8}$$

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DID: Identification

- With the parallel trend assumption, we can identify the policy effect δ by running:

$$Y_{ist} = \gamma_s + \lambda_t + \delta D_{st} + c_{ist} \tag{9}$$

- First difference: For same state, dif across time

$$E[Y_{ist}|s = PA, t = Nov] - E[Y_{ist}|s = PA, t = Feb] = \lambda_{Nov} - \lambda_{Feb} \tag{10}$$
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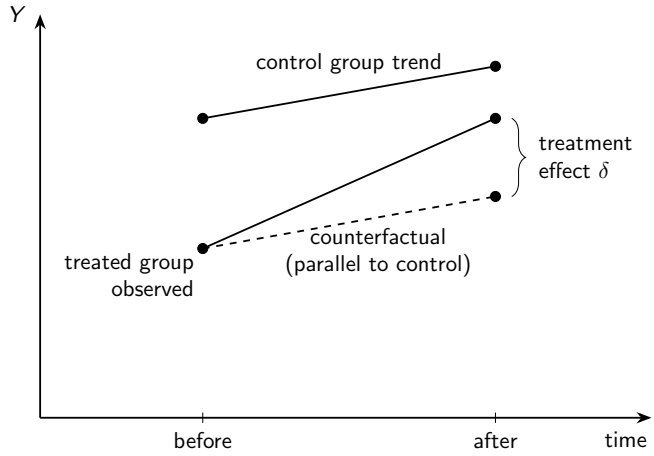
We are taking untreated group as the control!



DID: Test of Parallel Trend

- After the implementation of the policy at t_0, we can no longer observe Y_{0i} for the treated group
- Thus, we cannot test parallel trend after t_0
- We test parallel trend before t_0: Pre-trend test
- There are two simple ways to do that

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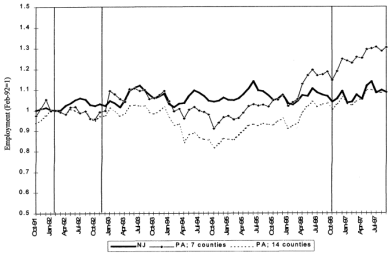

Figure 5.2.2: Employment in New Jersey and Pennsylvania fast-food restaurants, October 1991 to September 1997 (from Card and Krueger 2000). Vertical lines indicate dates of the original Card and Krueger (1994) survey and the October 1996 federal minimum-wage increase.

Source: Angrist and Pischke (2009), Figure 5.2.2, from Card and Krueger (2000)

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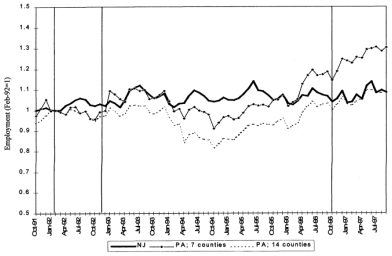

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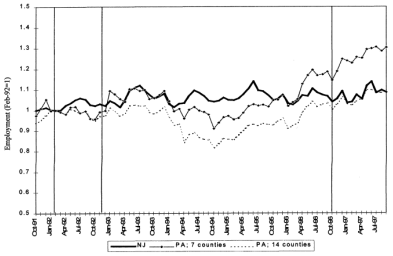

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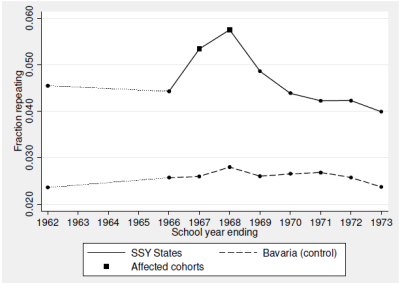

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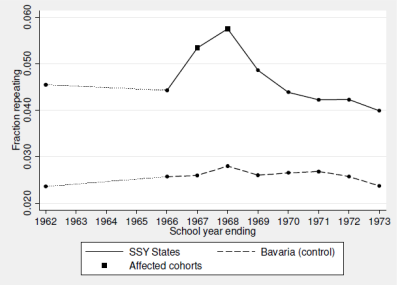

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2. Event Study Regression
   - If we have data from -T to T', and the policy D_{it} is implemented at $t = 0$
   - Let D_s be the dummy of whether in the treated group
   - Run the following regression

$$Y_{ist} = \gamma_s + \lambda_t + \sum_{\tau \neq -1} 1(t = \tau)\delta_\tau + \epsilon_{ist} \tag{13}$$

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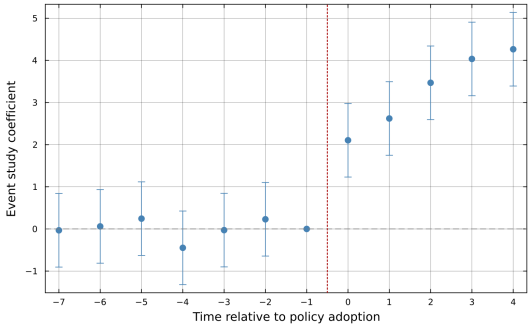
- Then we draw the changes of $\hat{\alpha}$ for each time period and have:



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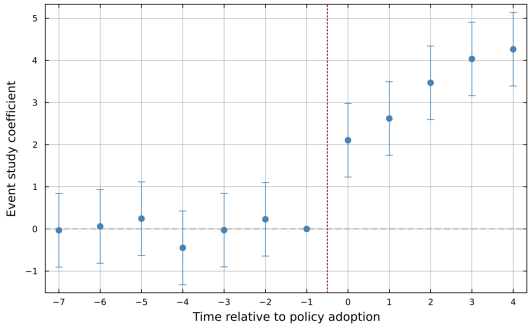

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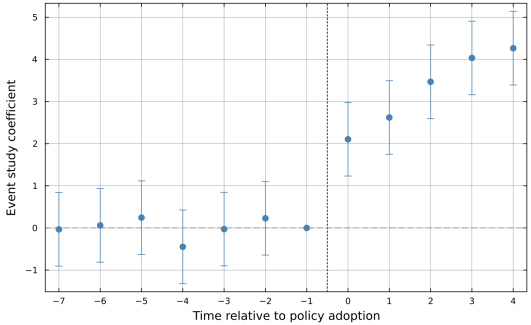

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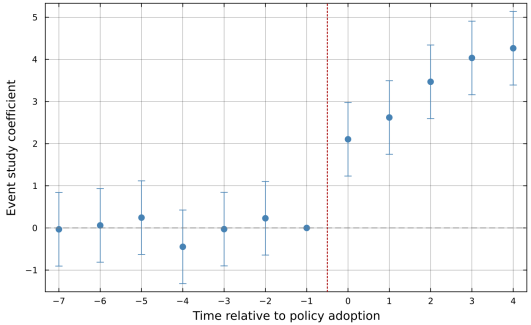

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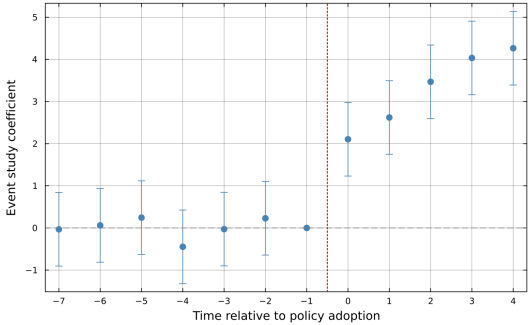

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- When you are doing causal research, a central question is: What kind of variations are used to identify the causal effect?
- It is very very very important!!!
- It determines how you can interpret your results
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Identification Source

- It becomes complicated in panel data $\Leftarrow$ more dimensions
- Sometimes, people control many FEs at different levels
- Some are even combined with IV, RD, or other regression structure
- Still, you should always be very clear about your identification source
- Let's consider a simple case: effects of working experience on wage
- For individual i from family j at time t:

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- When controlling for individual FE, you are using variations across time (t level) for the same people
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- We have introduced basic methods of causal inference in panel data
- Now we go to four important extensions
  - Recent development in pre-trend testing
  - Synthetic Control Method: When you do not have parallel trend
  - Triple differences (DDD)?
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- Is event study a perfect tool to test parallel pre-trend?
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1. Statistical power is low: Likely to have type-II error
   - Pre-existing trends that produce meaningful bias may not be detected
   - Assuming a linear violation of parallel trend: $\delta_{1t} - \delta_{0t} = \gamma t$ for some slope γ
   - Roth implements some Monte Carlo Simulation using data from 70 papers
   - He finds that if you want to detect this violation for 80% of the time, the bias has to be as large as the estimated TE! (100% bias)
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- Type I error: H0 is true but we reject it, α
- Type II error: H0 is false but we do not reject it, β
- Significance level: Probability of committing Type I error, α
- Power: Probability of rejecting H0 if it is false, $1 - \beta$
- Power means the power to reject a false H0

Null hypothesis is . . .	True	False
Rejected	Type I error False positive Probability $= \alpha$	Correct decision True positive Probability $= 1 - \beta$
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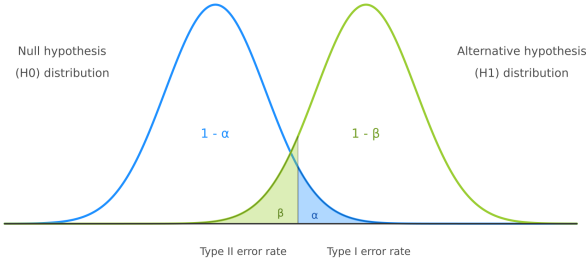
Tradeoff!!!

- Now you have to choose a threshold critical value to make your rejection decision
- Go left, you have larger α; Go right, you have larger β



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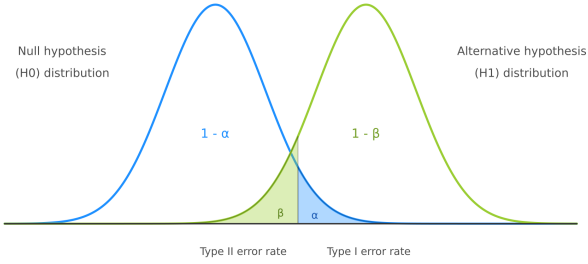

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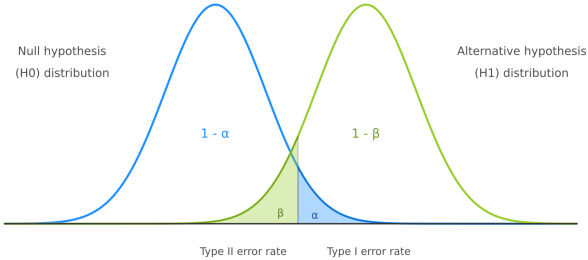

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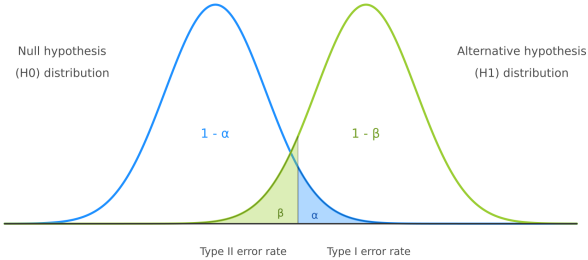

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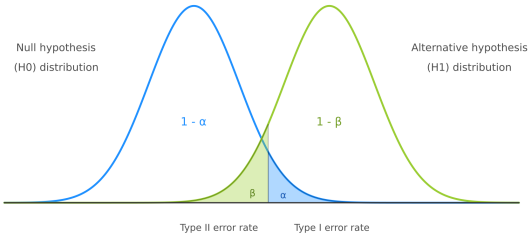

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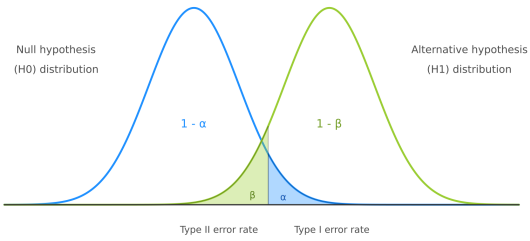

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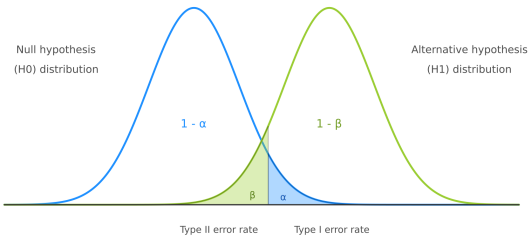

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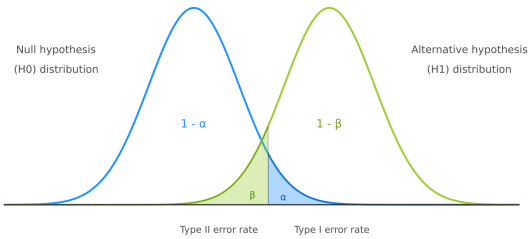

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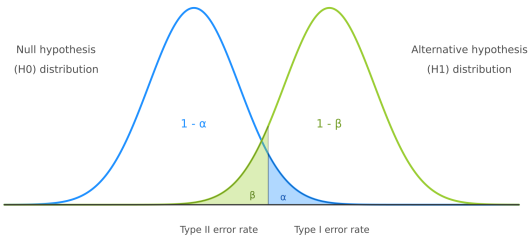

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- In traditional testing, we try to be conservative about rejecting H0
- Minimize Type I error probability α to be smaller than some level (10%, 5%, 1%)
- It then leads to large β! $\Longrightarrow$ small power
- But in pre-trend testing, actually we care more about power
- We want to be more conservative about NOT rejecting H0
- If you make rejection very hard, of course it is very likely that you have good pre-trend test result

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2. By selecting samples that can pass the test
   - 2a. Underestimate the variance of the estimation
   - 2b. If there is bias, conditioning on passing the event study test may exacerbate it (Adding bias to point estimation)
     - The bias is certainly exacerbated in common cases (monotone trends and homoscedastic errors)
     - Thus, the effect of pre-trend testing can be ambiguous
     - Recent work (Roth 2022) and Rambachan (2023) are interesting

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       Reject non-parallel cases (good) vs. Increasing bias if there is bias (bad)

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Pre-trend Testing: New Development

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Pre-trend Testing: New Development

Practical suggestions proposed by Roth

- Most important advice:
  Always use your economic knowledge to verify the parallel trend assumption!
- Do not think you are safe when event study does not detect anything
- Do power calculations against economically relevant violations of parallel trends
  (Stata package `pretrends`)
- If you know the functional form of the differences in trends, control it
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Synthetic Control: Main Idea

- The critical assumption for DID is parallel trend
- What if we do not have it? What if no control unit trends in parallel with the treated unit?
- Often, the treated unit is just one aggregate entity
  For example, one province implements a new policy
- Then there is no clean control group
- Let's create one control group! $\Longrightarrow$ Synthetic Control
- Synthetic control is a matching method

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All the following content comes from Abadie, Diamond, and Hainmueller (2010, 2015); Abadie (2021)

- The synthetic control method is based on the idea that, when the units of observation are a small number of aggregate entities, a combination of unaffected units often provides a more appropriate comparison than any single unaffected unit alone.

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Synthetic Control: Main Idea

- Take Abadie, Diamond, and Hainmueller (2010) as an example
- California implemented Proposition 99 in 1988
- It is a large-scale tobacco control program
  - A 25-cent per pack excise tax on the sale of tobacco cigarettes, cigars and chewing tobacco
  - A ban on cigarette vending machines in public areas accessible by juveniles

Synthetic Control: Main Idea

- But it seems that pre-trends are very different across states

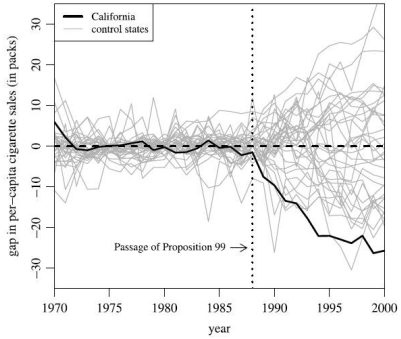

Figure 5. Per-capita cigarette sales gaps in California and placebo gaps in 34 control states (discards states with pre-Proposition 99 MSPE twenty times higher than California's).

Source: Abadie, Diamond, and Hainmueller (2010)

Synthetic Control: Main Idea

- Even when you average over all control states, you have this

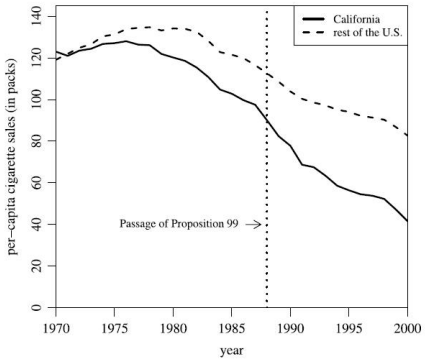

Figure 1. Trends in per-capita cigarette sales: California vs. the rest of the United States.

Source: Abadie, Diamond, and Hainmueller (2010)

Synthetic Control: Main Idea

- Then you have to combine them to create a "synthetic" control state
- A man-made "synthetic" California

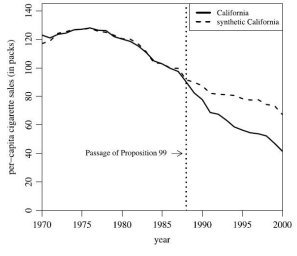

Figure 2. Trends in per-capita cigarette sales: California vs. synthetic California.

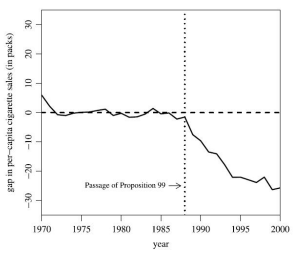

Figure 3. Per-capita cigarette sales gap between California and synthetic California.

(a) (b)

Source: Abadie, Diamond, and Hainmueller (2010)

Synthetic Control: Settings

- Suppose we have $j = 1, 2, ..., J + 1$ units (provinces, cities...), spanning T periods
- Periods $1, ..., T_0$ are pre-treatment and treatment starts at $T_0 + 1$; $j = 1$ is the treated unit
- We call $j = 2, 3, ..., J + 1$ as "donor pool"
- We will create the synthetic control group from units in this "donor pool"
- X_{kj} are observed characteristics, which can include pre-treatment values of Y
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Synthetic Control: Settings

- Define potential outcome: Y_{jt}^I, Y_{jt}^N
- Treatment effect of interest: $\tau_{1t} = Y_{1t}^I - Y_{1t}^N$ for $t > T_0$
- Treatment effect can vary across time
- A synthetic control is defined as a weighted average of the units in the donor pool:

$$\hat{Y}_{1t}^N = \sum_{j=2}^{J+1} w_j Y_{jt}$$

- w_j is the weight assigned to donor j, with $w_j \geq 0$ and $\sum_{j=2}^{J+1} w_j = 1$
- Then we can estimate the treatment effect:

$$\hat{\tau}_{1t} = Y_{1t} - \hat{Y}_{1t}^N$$

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Synthetic Control: Settings

- How to define the weights?
- Assume that each unit has k observed characteristics (predictors), indexed by $h = 1, 2, ..., k$
- We minimize the following:

$$\|X_1 - X_0 W\| = \left(\sum_{h=1}^{k} v_h (X_{h1} - w_2 X_{h2} - ... - w_{J+1} X_{h,J+1})^2\right)^{1/2} \quad (15)$$

- This is the weighted Euclidean distance between X_1 and X_0
- We try to find a combination of donors that can mimic our treated group the best
- Watch out: the difference between weights v and weights w

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Synthetic Control: Procedure

How to determine v?

- Step 1: Divide all pre-treatment sample into 2 parts
  Training: $t = 1, ..., t_0$; and Validation: $t = t_0 + 1, ..., T_0$
- Step 2: For every possible V, calculate w by minimizing equation (15) using training data $\Longrightarrow \tilde{w}(V)$
- Step 3: Find the best V^* that minimizes MSPE using validation data:

$$\frac{1}{T_0 - t_0} \sum_{t=t_0+1}^{T_0} \left(Y_{1t} - \tilde{w}_2(V)Y_{2t} - ... - \tilde{w}_{J+1}(V)Y_{J+1,t}\right)^2$$

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$$\frac{1}{T_0 - t_0} \sum_{t=t_0+1}^{T_0} (Y_{1t} - \tilde{w}_2(V) Y_{2t} - ... - \tilde{w}_{J+1}(V) Y_{J+1,t})^2$$

- Step 4: Recompute the final unit weights $W^* = \tilde{w}(V^*)$ using the validation data
- Avoid overfitting by separating training from validation (Chapter 3)

Synthetic Control: Procedure

How to determine v?

- Step 1: Divide all pre-treatment sample into 2 parts
  Training: $t = 1, ..., t_0$; and Validation: $t = t_0 + 1, ..., T_0$
- Step 2: For every possible V, calculate w by minimizing equation (15) using training data $\Rightarrow \tilde{w}(V)$
- Step 3: Find the best V^* that minimizes MSPE using validation data:

$$\frac{1}{T_0 - t_0} \sum_{t=t_0+1}^{T_0} \left(Y_{1t} - \tilde{w}_2(V) Y_{2t} - ... - \tilde{w}_{J+1}(V) Y_{J+1,t}\right)^2$$

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Synthetic Control: An Example

- How to estimate the effect of the 1990 German reunification
- Treated: West Germany; Untreated: Other OECD countries

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Synthetic Control: An Example

- v are weights for economic predictors: The importance of each predictor for the match procedure

TABLE 1
ECONOMIC GROWTH PREDICTOR MEANS BEFORE THE GERMAN REUNIFICATION

	West Germany (1)	Synthetic West Germany (2)	OECD average (3)	Austria (nearest neighbor) (4)
GDP per capita	15,808.9	15,802.2	13,669.4	14,817.0
Trade openness	56.8	56.9	59.8	74.6
Inflation rate	2.6	3.5	7.6	3.5
Industry share	34.5	34.4	33.8	35.5
Schooling	55.5	55.2	38.7	60.9
Investment rate	27.0	27.0	25.9	26.6

Source: Abadie, Diamond, and Hainmueller (2015)

Synthetic Control: An Example

- w are weights for compared countries: the importance of each country in forming the synthetic Germany

TABLE 2
SYNTHETIC CONTROL WEIGHTS FOR WEST GERMANY

Australia	—
Austria	0.42
Belgium	—
Denmark	—
France	—
Greece	—
Italy	—
Japan	0.16
Netherlands	0.09
New Zealand	—
Norway	—
Portugal	—
Spain	—
Switzerland	0.11
United Kingdom	—
United States	0.22

Source: Abadie, Diamond, and Hainmueller (2015)

Synthetic Control: An Example

- Synthetic West Germany

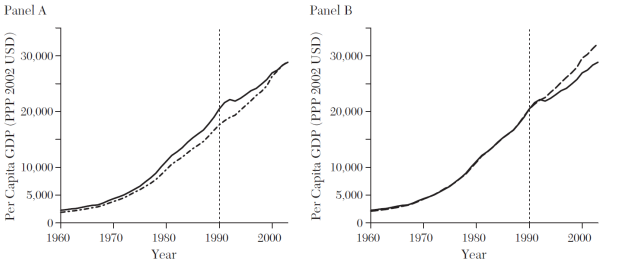

Figure 1 Synthetic Control Estimation in the German Reunification Example

Notes: Panel A compares the evolution of per capita GDP in West Germany to the evolution of per capita GDP for a simple average of OECD countries. In panel B the comparison is with a synthetic control calculated in the manner explained in subsection 3.2. See Abadie, Diamond, and Hainmueller (2015) for details.

Source: Abadie, Diamond, and Hainmueller (2015)

Synthetic Control: Conclusion

Several things you should remember for synthetic control

- Post-treatment outcomes cannot be used in X! (but pre-treatment outcomes can)
- There can be bias if there are unobserved endogenous characteristics that cannot be matched
- When the difference $X_1 - X_0 W^*$ is large (pre-treatment fit is bad), do not use synthetic control
- If the size of the donor pool is too large, there can be over-fitting
- So, choose the donor pool judiciously

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Triple Differences (DDD)

- Another method to solve the unparallel trend is triple differences (DDD)
- Assume that in Card and Krueger (1994), PA and NJ can have different time trends in the average wage due to some unobserved state-specific shock
- However, minimum wage only affects cashier at McDonald's, but not teachers
- The wage gap between cashiers and teachers has the same trend in PA and NJ
- So the only treated group is cashier in NJ
- The control group includes cashiers and teachers in PA, and teachers in NJ

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Triple Differences (DDD): 3WFE

- Denote i as individual, g as occupation group ($g = 1$ if cashier), s as state ($s = 1$ if NJ), t as time ($t = 1$ if Nov)
- We can extend the TWFE to have

$$Y_{it} = \rho D_{it} + \lambda_{g,t} + \delta_{s,t} + \alpha_i + \epsilon_{it} \tag{16}$$

- This is called Three-way Fixed Effect Model (3WFE)
- We control for i-level (thus, g, s-level), g, t-level, and s, t-level FE
- $D_{it} = 1$ only if $t = 1, g = 1, s = 1$
- It can be easily understood as a difference between two DIDs
- $\delta_{s,t}$ absorbs exactly the differential state trends that the two-state DID could not control
- Remaining assumption: no shock other than the policy hits the specific triple cell (cashiers in NJ after the policy)

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DDD: Identification

- Here, the extended parallel trend assumption is that $E[\epsilon_{it}|\lambda_{g,t}, \delta_{s,t}, \alpha_i, D_{it}] = 0$
- With the extended parallel trend assumption, we can identify the policy effect δ:
- First difference: For same state, same occupation, dif across time (α_i is canceled)

$$E[Y_{it}|s=0, g=0, t=1] - E[Y_{it}|s=0, g=0, t=0] = \lambda_{0,1} - \lambda_{0,0} + \delta_{0,1} - \delta_{0,0} \quad (17)$$
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- Second difference: For same state, dif in trends across occupation

$$(18) - (17) = \lambda_{1,1} - \lambda_{1,0} - (\lambda_{0,1} - \lambda_{0,0}) \quad (21)$$
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- Second difference: For same state, dif in trends across occupation

$$(18) - (17) = \lambda_{1,1} - \lambda_{1,0} - (\lambda_{0,1} - \lambda_{0,0}) \quad (21)$$

$$(20) - (19) = \lambda_{1,1} - \lambda_{1,0} - (\lambda_{0,1} - \lambda_{0,0}) + \rho \quad (22)$$

- Third difference: Difference across state

$$(22) - (21) = \rho \quad (23)$$

Conclusion

- When we have data across time for different units, we have panel data
- FE, dummy, and FD regressions can cancel out time-invariant confounders
- FE and dummy regressions are identical
- FE and FD are identical in 2-period cases, but different for more than 2
- FE is more efficient if errors are not correlated; FD is more efficient if errors are random walks

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- The main assumption for DID is parallel trend
- Traditionally, we validate parallel trend assumption in two steps:
  1. Draw a figure
  2. Run event study
- However, statistical test minimizes T1ER, which inflates T2ER
- Be careful using them!
- Check the power and validate your assumption using economic knowledge

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- When it is hard to find a good control group with parallel trend
- You can create one using synthetic control method
- You assign two sets of weights and take the weighted average
  - Weights for each characteristics
  - Weights for each donor unit
- Then you create a control group to mimic the behavior of the treated group
- Or you can use DDD if there is a further layer of difference which gives you the exogeneity

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