Frontier Topics in Empirical Economics: Week 12
Discrete Choice Model I

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Introduction: Discrete Choice Model

- In previous lectures, we focus on reduced-form approach
- In this lecture, we will give a very brief introduction to the Discrete Choice Model
- It considers problems when y is discrete
- DCM stays in the intersection of reduced-form and structural models
- It is an important method for both approaches

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Introduction: Discrete Choice Model

- You can learn and understand it in both frameworks
- If you understand it in a reduced-form way
  - Another kind of non-linear regression model
  - Harder to interpret, but better than LPM to fit when y is binary
- If you understand it in a structural way, it is actually a brand new world
  - Each parameter is a structural parameter of the behavior model
  - There is underlying welfare implication

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Motivating Example: Female Labor Participation

Still remember the example in our first class?

- Consider a female labor participation problem
- Utility maximization of the female i:

$$\begin{aligned} max \quad & U_i(c_i, 1 - l_i) + \epsilon_{il} \\ s.t. \quad & c_i = w_i l_i \end{aligned} \tag{1}$$

c_i: consumption; l_i: labor supply; ϵ_{il}: unobserved taste shock; w_i: wage

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- Assume that l_i is binary (work, not work)
- $l_i = 1$ if $U(l = 1) \geq U(l = 0)$:

$$U_i(w_{i},, 0) + \epsilon_{i1} \geq U_i(0, 1) + \epsilon_{i0} \quad (2)$$

- Then given w_i, we have a threshold value of $\epsilon_{i1} - \epsilon_{i0}$ to have i to choose to work:

$$l_i = 1 \quad \text{if} \quad \epsilon_{i0} - \epsilon_{i1} < \epsilon^* \quad (3)$$

$$\epsilon^* = U_i(w_i, 0) - U_i(0, 1)$$

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- Assume that shock $\epsilon_{i1} - \epsilon_{i0}$ has a CDF $F_{\epsilon|w}$
- We have the following working probability for i:

$$
\begin{aligned}
G(w) = Pr(I = 1|w) &= \int_{-\infty}^{\epsilon^*} dF_{\epsilon|w} \\
&= F_{\epsilon|w}(\epsilon^*(w)) \qquad (4)
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- Two empirical research approaches for this question

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- DCM describes decision makers' choices among discrete alternatives
- A man chooses whether to smoke or not
- A student chooses how to go to school (Bus/Taxi/Bike)
- A firm chooses whether to enter a local market (Walmart vs. Local store)

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- Assume that we have N decision makers, choosing among a set of J alternatives $1, 2, ..., j$
- Decision maker n can get utility U_{nj} for choosing j
- The optimization is: n choose i if and only if

$$U_{ni} > U_{nj}, \forall j \neq i \quad (5)$$

- Researcher does not observe utility directly
- We see their choice results (revealed preference)
- We observe attributes of choices faced by agents x_{nj}, and agents' personal characteristics s_n
- Thus, we denote $V_{nj} = V(x_{nj}, s_n)$ as representative utility

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- Utility of choice j to agent n can be expressed as:

$$U_{nj} = V_{nj} + \epsilon_{nj} \tag{6}$$

- ϵ_{nj} is the part of utility affected by unobserved factors
- Assume that we have pdf $f(\epsilon_n)$ for $\epsilon_n^{'} = [\epsilon_{n1}, ...\epsilon_{nJ}]$ across the population

$$\begin{aligned} P_{ni} &= P(U_{ni} > U_{nj}, \forall j \neq i) \\ &= P(V_{ni} + \epsilon_{ni} > V_{nj} + \epsilon_{nj}, \forall j \neq i) \\ &= P(\epsilon_{nj} - \epsilon_{ni} < V_{ni} - V_{nj}, \forall j \neq i) \\ &= \int_{\epsilon} I(\epsilon_{nj} - \epsilon_{ni} < V_{ni} - V_{nj}, \forall j \neq i) f(\epsilon_n) d\epsilon_n \end{aligned}$$

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- Different assumptions of the pdf $f(\epsilon_n)$ gives different models
- This expression does not guarantee a closed-form choice probability
- Type I Extreme Value Distribution gives Logit (Closed-form)
- Normal Distribution gives Probit (Not closed-form)
- Logit and Probit are specific types of DCM

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Introduction to DCM: Identification

- The identification of the DCM is important
- It relates to some primitive properties of utility function
- It can be concluded in two statements
  - 1. Only differences in utility matter
  - 2. The scale of utility is arbitrary
- Why is this the case?
- Let's go back to the fundamental theory of utility

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- The identification of the DCM is important
- It relates to some primitive properties of utility function
- It can be concluded in two statements
  - 1. Only differences in utility matter
  - 2. The scale of utility is arbitrary
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- Utility function comes from preference
- Assume that we have goods set X, a preference relation $\succsim$ defined on X, satisfying
  - (1)Completeness: $\forall x, y \in X$, we have $x \succsim y$ or $y \succsim x$ (or both)
  - (2)Transitivity: $\forall x, y, z \in X$, if $x \succsim y$, $y \succsim z$, then $x \succsim z$
- We call it a "rational" preference

Definition 1.B.2 in MWG

A function $u : X \to R$ is a utility function representing preference $\succsim$ if $\forall x, y \in X$, $x \succsim y \Leftrightarrow u(x) \geq u(y)$

- There exists a utility function $\Rightarrow$ Preference is rational

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- A utility function assigns a numerical value to each element in X in accordance with the individual's preferences
- Thus, utility is a representation of preference!
- Preference is ordinal $\Longrightarrow$ Utility is ordinal
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- 1. Only differences in utility matter
- 2. The scale of utility is arbitrary
- Let's use an example to reveal these two statements
- Assume that you can go to school either by bus (b) or by car (c)
- T_j is the speed of choice j, k_j is choice amenity

$$U_c = \alpha T_c + k_c + \epsilon_c$$
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$$U_c - U_b = \alpha(T_c - T_b) + (k_c - k_b) + (\epsilon_c - \epsilon_b)$$

   - Only $(k_c - k_b)$ can be identified, but not k_c and k_b separately
   - System u_j and $u_j + 1$ are observational equivalent
   - I don't care it is $u_i - u_j$ or $u_i + 1 - (u_j + 1)$
   - Thus, you cannot give each alternative a constant
   - What to do in practice: Normalize the utility of one of the alternatives to be zero (Implicitly done by running logit/probit regressions)

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   - In addition, not all differences matter
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$$U_{nc} = \alpha T_c + \beta Y_n + \gamma Y_n T_c + \epsilon_{nc}$$
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$$U_{nb} - U_{nc} = \alpha(T_b - T_c) + \gamma Y_n(T_b - T_c) + (\epsilon_{nb} - \epsilon_{nc})$$

   - Y_n is canceled out, only γ is identified, but not β
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2. The scale of utility is arbitrary
   - Similarly, u_j and $u_j * 2$ are observational equivalent
   - I don't care it is $u_i - u_j$ or $2 * (u_i - u_j)$
   - Assume that we have the following model 1

$$U_{nc} = \alpha T_c + \beta Y_n + \epsilon_{nc}$$
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   - And the following model 2

$$2U_{nc} = \alpha 2 T_c + 2\beta Y_n + 2\epsilon_{nc}$$
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   - They are observational equivalent

Introduction to DCM: Identification

2. The scale of utility is arbitrary
   - Similarly, u_j and $u_j * 2$ are observational equivalent
   - I don't care it is $u_i - u_j$ or $2 * (u_i - u_j)$
   - Assume that we have the following model 1

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   - Thus, we need to normalize the scale
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Introduction to Logit Model: Settings

- Assume that ϵ_{nj} is i.i.d. Type One Extreme Value (T1EV)
- PDF: $f(\epsilon_{nj}) = e^{-\epsilon_{nj}} e^{-e^{-\epsilon_{nj}}}$
- CDF: $F(\epsilon_{nj}) = e^{e^{-\epsilon_{nj}}}$
- Since error terms are independent, we have: $F(\epsilon_{n1}, ..., \epsilon_{nJ}) = e^{\sum_{j=1,...,J} e^{-\epsilon_{nj}}}$
- Then we call this DCM a Logit model

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Introduction to Logit Model: Choice Probability

- Let's derive the choice probability of Logit model

$$\begin{aligned} P_{ni} &= P(U_{ni} > U_{nj}, \forall j \neq i) \\ &= \int_{\epsilon} I(\epsilon_{nj} - \epsilon_{ni} < V_{ni} - V_{nj}, \forall j \neq i) f(\epsilon_n) d\epsilon_n \end{aligned}$$

- It turns out that we can write the (multinomial) choice probability as:

$$P_{ni} = \frac{e^{V_{ni}}}{\sum_j e^{V_{nj}}} \tag{7}$$

- Usually, we have to normalize one of the choices (let's say, choice j_0) to have a utility of zero:

$$P_{ni} = \frac{e^{V_{ni}}}{1 + \sum_{j \neq j_0} e^{V_{nj}}} \tag{8}$$

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- Homework: Derive the choice probability equation (7). The answer is in Train's book, Chapter 3.

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- What does this choice probability mean?

$$P_{ni} = \frac{e^{V_{ni}}}{\sum_j e^{V_{nj}}}$$

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- Compatible with choice probability definition: $0 < P_{ni} < 1$, $\sum_i P_{ni} = 1$ (Not like LPM)

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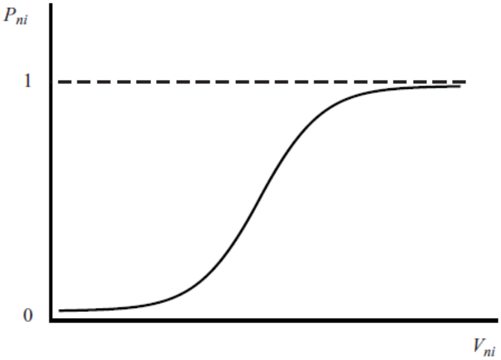
Introduction to Logit Model: Choice Probability

- The relation of probability with representative utility is sigmoid (S-shaped)



- Marginal effects of V_{ni} on P_{ni} increase first and then decrease
- If you use a linear fit, which part do you fit the best?

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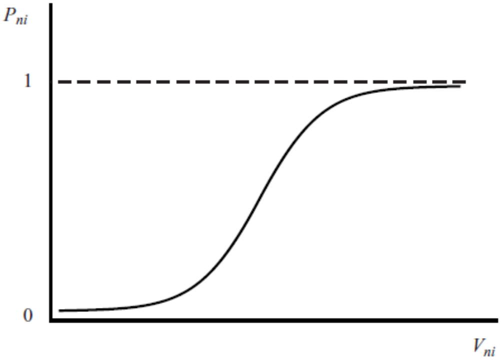

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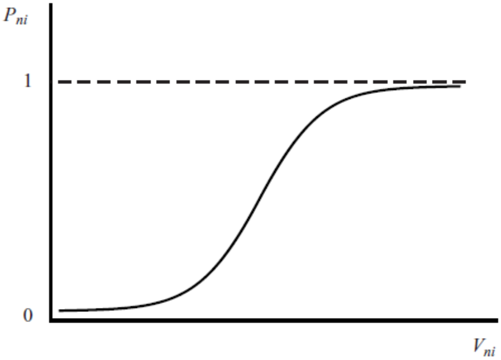

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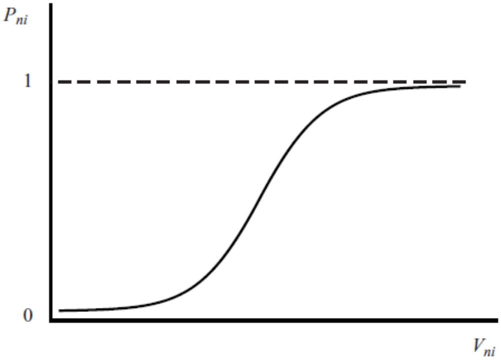

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Introduction to Logit Model: IIA

- An important property: Independence from Irrelevant Alternatives (IIA)
- IIA: For any two alternatives i, k, the ratio of the logit probability is

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- The ratio has nothing to do with other alternatives
- Prob ratio between any pair of choices depends only on their own choice values
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- A manifestation of IIA is proportionate shifting
- A change in an attribute z of choice j, will change probabilities of all other choices by the same proportion
- With linear utility, the elasticity of choice prob i on changes in z of choice j is

$$E_{iz_{nj}} = \frac{\partial P_{ni}}{\partial z_{nj}} \frac{z_{nj}}{P_{ni}} = -\beta_z z_{nj} P_{nj}, \forall i$$

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- It can save computational resources when the number of choices is large
- But it is also limited: Red bus-Blue bus problem
- We will introduce more flexible models soon

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- The derivative of choice probability on its own attribute is:

$$\frac{\partial P_{ni}}{\partial z_{ni}} = \frac{\partial V_{ni}}{\partial z_{ni}} P_{ni}(1 - P_{ni}) \tag{10}$$

- Parameter is not marginal effect: $\frac{\partial P_{ni}}{\partial z_{ni}} \neq \frac{\partial V_{ni}}{\partial z_{ni}}$
- Even if V is linear, you cannot interpret $\beta = \frac{\partial V_{ni}}{\partial z_{ni}}$ as marginal effect of z on P
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Introduction to Logit Model: Derivatives and Marginal Effect

- Homework 2: Derive equation 10. The answer is in Train's book, Chapter 3.

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- We are usually interested in the overall welfare of a consumer
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Nested Logit

- In the previous case, we assume that alternatives are at the same level
- What if they have a hierarchy structure?
- Now let's consider a more general model called nested logit

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- As we have shown, Logit has a property of IIA
- Given two options A and B, changes of the third option would not change the relative probability of A and B
- In some situations, this property is not plausible

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- Assume that we have two choices
  Blue Bus vs. Taxi
- $P_{BB} = P_T = \frac{1}{2}$
- One day, the bus company decides to introduce some buses with a new color, red
- Now we have blue bus, red bus, taxi
- Red/blue bus is identical besides their color $\Longrightarrow P_{RB} = P_{BB}$
- Due to IIA, we have: $P_{RB} = P_{BB} = P_T = \frac{1}{3}$
- You increase the probability of choosing bus by basically doing nothing

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Nested Logit: Setting

- To solve the Blue/Red bus issue, we introduce an extension of Logit model: Nested Logit Model
- We allow for correlations over some of the options
- We have utility of choice j to agent n can be expressed as:

$$U_{nj} = V_{nj} + \epsilon_{nj} \tag{11}$$

- In nested logit, we have $\epsilon = (\epsilon_{n1}, ..., \epsilon_{nJ})$ are jointly distributed as a generalized extreme value (GEV)

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- Let the choice set be partitioned into K subsets $B_1, ..., B_K$ called nests
- CDF of $\epsilon = (\epsilon_{n1}, ..., \epsilon_{nJ})$ is:

$$F(\epsilon) = exp(-\sum_{k=1}^{K}(\sum_{j\in B_k} e^{-\frac{\epsilon_{nj}}{\lambda_k}})^{\lambda_k})$$

- Marginal distribution of each ϵ_{nj} is univariate T1EV
- Any two options within the same nest, have correlated ϵ
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- We can show that the choice probability of nested logit is:

$$P_{ni} = \frac{e^{V_{ni}/\lambda_k} (\sum_{j \in B_k} e^{V_{ni}/\lambda_k})^{\lambda_k - 1}}{\sum_{l=1}^{K} (\sum_{j \in B_l} e^{V_{nj}/\lambda_l})^{\lambda_l - 1}} \tag{12}$$

- We have $(\sum_{j \in B_k} e^{V_{nj}/\lambda_k})^{\lambda_k - 1}$ in the numerator (All choices in the same nest)
- Given two alternatives $i \in k$ and $m \in l$, we have the probability ratio as:

$$\frac{P_{ni}}{P_{nm}} = \frac{e^{V_{nj}/\lambda_k} (\sum_{j \in B_k} e^{V_{nj}/\lambda_k})^{\lambda_k - 1}}{e^{V_{nm}/\lambda_l} (\sum_{j \in B_l} e^{V_{nj}/\lambda_l})^{\lambda_l - 1}}$$

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- If $k = l$, we have IIA for two choices in the same nest

$$\frac{P_{ni}}{P_{nm}} = \frac{e^{V_{ni}/\lambda_k}}{e^{V_{nm}/\lambda_l}}$$

- If $k \neq l$, we do not have IIA for two choices in different nests
- Relative probability of i, m is related to other choices in their own nests k and l
- But not choices in other nests
- We call it "Independence from Irrelevant Nests" (IIN)

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Nested Logit: An Example

- Auto=(Auto alone, Carpool), Transit=(Bus, Rail)

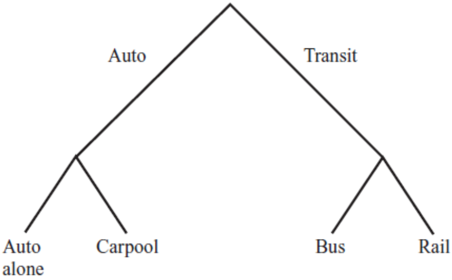

Figure 4.1. Tree diagram for mode choice.

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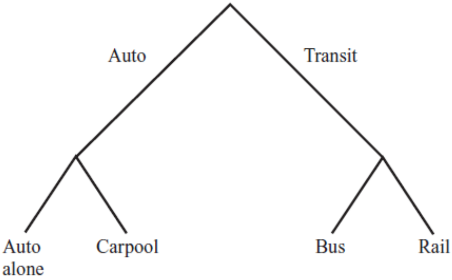

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Logit or LPM?

- An important practical question is, when to use Logit? When to use linear probability model (LPM)?
- Let's first list pros and cons
- For Logit: non-linear fitting with functional form assumption
  - Coefficients are "structural" and primitive ⇒ Utility, Production.
  - But coefficients are neither marginal effects nor weighted treatment effects
  - Computationally intensive, especially MLE for high-dimensional dummies
- For LPM: linear fitting, more an approximation
  - Coefficients are marginal effects, very easy to interpret
  - But will predict probability > 1 or < 0
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Here are some personal views

- If you do care about the primitive parameter $\Longrightarrow$ Logit
- If you are interested in extrapolating your prediction (predict y for x with few samples nearby) $\Longrightarrow$ Logit
- If you have x distributed pretty uniformly over the range, while want to predict y for very small or very large x $\Longrightarrow$ Logit
- Otherwise, you can choose LPM

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Conclusion: Main Takeaways

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- Logit is intrinsically a structural approach, whose parameters have structural meaning
- Logit is a special kind of DCM when the error is T1EV distributed
- Logit is convenient since it has closed-form choice probability and expected utility
- Logit has a property of IIA, that the relative probability of two choices is not affected by the third one
- The interpretation of Logit (or in general, non-linear model) is not as straightforward as Linear probability model

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- Nested Logit is a more general model than Logit
- We assume GEV: choices within the same nest have correlated ϵ
- IIA for two choices within the same nest but not across different nests
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