Frontier Topics in Empirical Economics: Week 10
Regression Discontinuity Design

Zibin Huang [1]

[1] College of Business, Shanghai University of Finance and Economics

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Introduction

- Assume that we want to examine the education quality of PKU and FDU
- The average wage for PKU graduates is 200,000 RMB/year
- The average wage for FDU graduates is 150,000 RMB/year
- Does this mean that PKU results in higher human capital growth than FDU?

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- Self-selection is always a problem in economic research
- Is school A more efficient than school B?
- Or just because they admit students with better initial quality?
- How to deal with this issue?

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- Of course you can always construct a selection model structurally
- But there is another design-based approach:
  Regression Discontinuity Design (RDD)
- The intuition for RDD is simple
- Draw PKU students just above the PKU admission line and FDU students just below it
- They are students who enroll in PKU/FDU by chance, thus, similar in ability
- Then compare their results

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Sharp RD

- Let's first consider a simple case: Sharp RD
- In Sharp RD, treatment rule is deterministic
- That is, you are definitely treated if you surpass the threshold
- Conversely, you are definitely not treated
- There is no uncertainty in treatment assignment

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- Suppose that we have treatment D_i determined by some x_i

$$D_i = \mathbf{1}\{x_i \geq x_0\} = \begin{cases} 1, & \text{if } x_i \geq x_0 \\ 0, & \text{if } x_i < x_0 \end{cases}$$

- x_i is called running variable
- x_0 is a known threshold or cutoff
- D_i is a deterministic function of x_i

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- In conventional matching, we compare samples with identical covariates
- In RD, we compare samples within a small neighborhood at treatment threshold

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- We can write a simple model for this RD

$$Y_i \quad f_0 \; x_i \; \mathbf{1} \; x_i \; \$ \; x_0 \quad f_1 \; x_i \; \mathbf{1} \; x_i \; ' \; x_0 \quad D_i \quad {}_i$$

- $f_0 \; x_i$ is the smoothing function below the threshold
- $f_1 \; x_i$ is the smoothing function above the threshold
- They are used to fit the trend far away from the cutoff
- D_i is the treatment indicator, jumping at $x_i \quad x_0$

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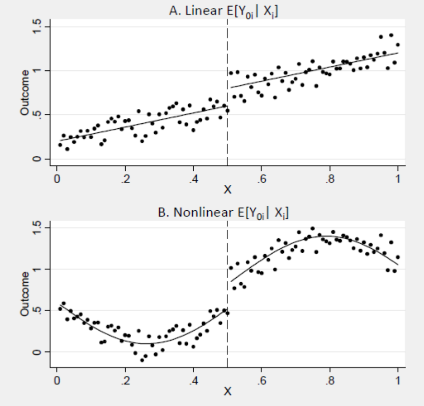

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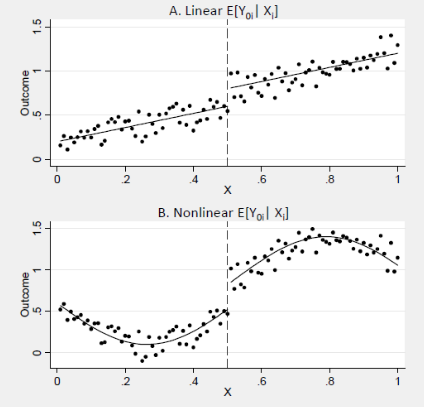

Sharp RD

- We can also use non-parametric and semi-parametric functions introduced in Week 2 lecture, which are more flexible
- The most recommended and commonly used one is the Local Linear/Quadratic Regression
- As we have discussed, there is a bias-variance tradeoff
- If you choose complicated smoothing function, you may lose your accuracy
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- But remember, effective sample size is usually limited in RD
- You are effectively using a small neighborhood around the cutoff
- So, do not use too complicated smoothing models
- Specifically, Gelman and Imbens (2019) claim that you should avoid using high-order polynomial (over third order)
  - It leads to noisy estimates (Runge's phenomenon)
  - RDD is very sensitive to the degree of the polynomial
  - Coverage of confidence intervals is smaller than nominal

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- An interesting example of Sharp RD is Lee (2008)
- What is the advantage for the party incumbency on reelection?
- Hard to identify since a party may have larger group of supporters for many reasons other than incumbency
- Blue state vs. Red state vs. Swing state

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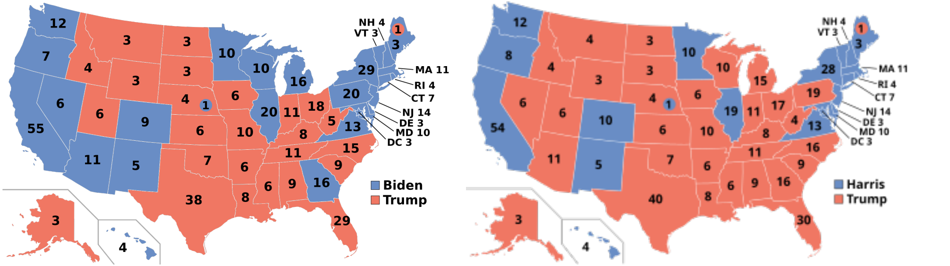
- Different parties are advantaged in different regions due to ideology, history, religion... reasons

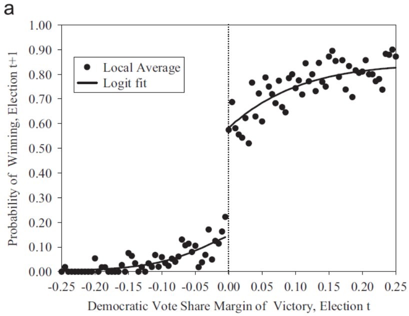


(a) 2020 (b) 2024

Figure: U.S. General Election Map

Sharp RD

- But for elections with very close results, winners and losers are similar
- Lee (2008) considers the probability of Democratic winning in regions where Democratic candidates won by small shares



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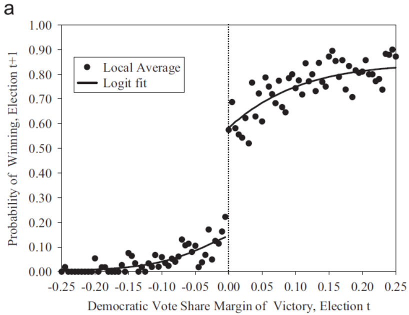

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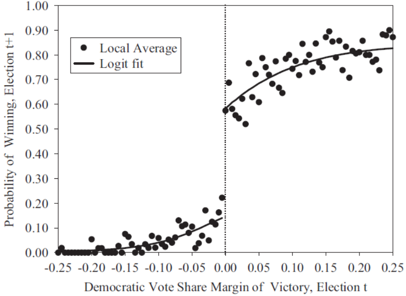

Fuzzy RD

- A more complicated case is Fuzzy RD
- In Fuzzy RD, treatment assignment is no longer deterministic
- There is uncertainty in being treated or not
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- Discontinuity in treatment probability, but not treatment

$$P(D_i = 1 | x_i) = \begin{cases} g_1(x_i) & \text{if } x_i \geq x_0 \\ g_0(x_i) & \text{if } x_i < x_0 \end{cases}; \text{ where } g_1(x_0) \neq g_0(x_0)$$

- Let's assume that $g_1(x_0) > g_0(x_0)$ (WLOG)
- Thus, surpassing the cutoff makes treatment more likely

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Fuzzy RD

- Denote $T_i = \mathbf{1}(x_i \geq x_0)$ as the indicator of whether passing the cutoff
- Then, we can naturally write Fuzzy RD as a 2SLS
- Treatment D_i is endogenous variable, cutoff indicator T_i is instrument
  - First stage: treatment D_i on cutoff indicator T_i
  - Second stage: outcome variable on first stage fitted value
- The smoothing function f should be included in both stages
- Very simple to implement RD in Stata: Packages such as *rdrobust*
- It helps you to implement bias-corrected CI with optimal bandwidth in Calonico, Cattaneo, and Titiunik (2014)
- You can also try optimal bandwidth in Imbens and Kalyanaraman (2012)

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Fuzzy RD

- Denote $T_i \quad \mathbf{1} \; x_i \; ' \; x_0$ as the indicator of whether passing the cutoff
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- And intuitively discussed its identification source
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Suppose that is fixed at . Further suppose that Assumptions (RD) and (A1) hold. We then have: $\frac{y^+ - y^-}{x^+ - x^-}$, where $y^+ = \lim_{z \to z_0^+} E[y_i | z_i = z]$ and $y^- = \lim_{z \to z_0^-} E[y_i | z_i = z]$

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- A1 is an exogeneity assumption; A2 is a no treatment sorting assumption
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Suppose that x_i is independent of β_i conditional on z_i near z_0. Further suppose that Assumptions (RD), (A1), and (A2) hold. We then have: $E[\beta_i | z_i = z_0] = \frac{y^+ - y^-}{x^+ - x^-}$

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- We invoke a set of assumptions similar to Imbens and Angrist (1994) on LATE

Assumption (A2) — Hahn, Todd, and Van der Klaauw (2001)

(i) $(\beta_i, x_i(z))$ is jointly independent of z_i near z_0. (ii) There exists $\epsilon > 0$ such that $x_i(z_0 + e) \geq x_i(z_0 - e)$ for all $0 < e < \epsilon$

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- Under exclusion restriction and monotonicity, we have:

Suppose that Assumptions (RD), (A1), and (A3) hold. We then have:

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- From this analysis of identification of RD
- We can derive what conditions we have to validate
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- The paper report this week is He, Wang, and Zhang (2020)
- It estimates the effect of environmental regulation on firm productivity in China
- The basic idea is very interesting
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- Thus, local gov officials enforce tighter environmental standards on firms just upstream rather than just downstream
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- An interesting extension of RDD is Regression Kink Design (RKD)
- Rather than using the discontinuity on treatment, we employ the kink on treatment
- The jump is no longer on the level, but the slope
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- In many countries, workers can get compensation when they are unemployed
- This is called unemployment benefit (UI)
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- If your last wage is too low, there is a minimum benefit level
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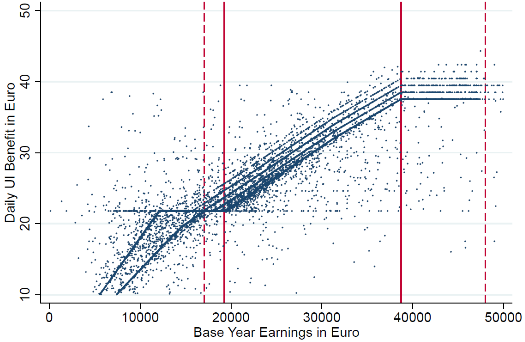
- Two kinks are noticeable: Minimum and Maximum

Figure 1: UI Benefits in 2004 Austria



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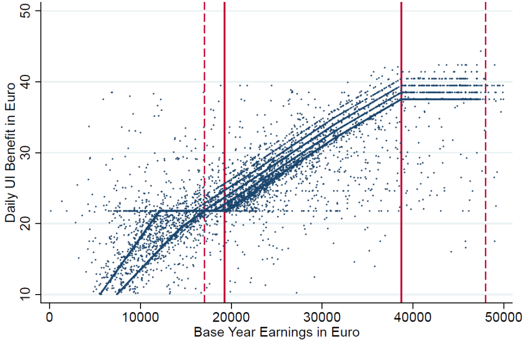

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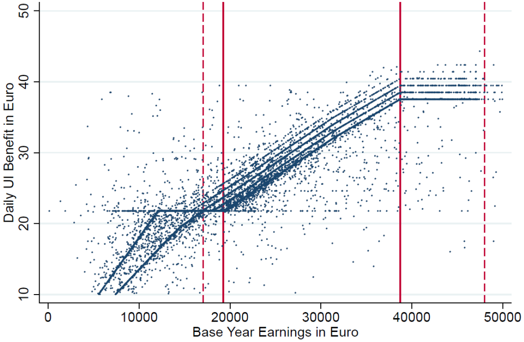

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- A controversial issue is that too generous UI can incentive workers not to search for new jobs
- It is important to investigate the relation between UI benefit B and unemployment duration Y
- Denote V as the wage of the last job, the running variable; U as an error term
- We have $Y \quad y \; B; V; U$ as the outcome function
- In a sharp kink design, B is a deterministic function of V: $B \quad b \; V$ with a slope change at $V \quad 0$
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- Assumption 1: (i) U is bounded; (ii) y is continuous and partially differentiable w.r.t. b and v, $y_b \; b; v; u$ is continuous (Regularity)
- Assumption 2: $y_v \; b; v; u$ is continuous around the kink $v \; 0$ (Exclusion). The kink exists only for $b \; v$, but not for the effect of v directly on y.
- Assumption 3: Treatment assignment rule $b \; v$ is known, continuous, and has a kink at $v \; 0$ (Kink existence)
- Assumption 4: Conditional density $f_{V\P U} \; v$ and its partial derivative w.r.t v are continuous around the kink $v \; 0$ (No kink for confounders)

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- Then we have the non-parametric identi cation of RKD

[illegible]

In a valid Sharp RKD, that is, when Assumptions [illegible] hold:

(a) $P(U \leq u | V = v)$ [illegible] is continuously differentiable in v [illegible] in the neighborhood of the kink.

(b) $E[Y_i | B_i = b_0, U, V = 0]$ [illegible]

- Sharp RKD is dividing slope change of Y ¶V by slope change of v
- On the contrary, RDD divides level by level
- Sharp RKD identi es the ATE for individuals with $B = b_0; V = 0$

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Proposition 1 in Card et al. (2015)

In a valid Sharp RKD, that is, when Assumptions 1-4 hold:
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- Sharp RKD is dividing slope change ofE Y ¶V by slope change of b v
- On the contrary, RDD divides level by level
- Sharp RKD identi es the ATE for individuals with B b$_0$; V 0

Extension of RDD: RKD

- Then we have the non-parametric identi cation of RKD

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(a) $P(U \leq u \mid V = v)$ is continuously differentiable in v at $v = 0$ $\forall u \in I_U$, where I_U is the neighborhood of the kink.

(b) $$E[y_b(b_0, 0, U) \mid V = 0] = \frac{\lim_{v \to 0^+} \frac{dE[Y \mid V = v]}{dv}\Big|_{v = v_0} - \lim_{v \to 0^-} \frac{dE[Y \mid V = v]}{dv}\Big|_{v = v_0}}{\lim_{v \to 0^+} \frac{db(v)}{dv}\Big|_{v = v_0} - \lim_{v \to 0^-} \frac{db(v)}{dv}\Big|_{v = v_0}}$$

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- The intuition here is as follows
- A change in the slope of treatment probability results in a change in the slope of average outcome
- If there is no change of slope for unobserved confounders
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- What about Fuzzy case?
- The result is very complicated, but with no surprising intuition
- In a Fuzzy RKD, we identify a LATE for individuals who have UI slope changes at the kink
- The larger you change, the larger weight you have
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Conclusion

- When you have a discontinuity in treatment, you can use RDD
- Sharp RDD is matching
  - Using samples around the cuto
  - It identi es ATT for individuals around the cuto
- Fuzzy RDD is IV
  - Using cuto indicator as instrument
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- It also identi es ATT and LATE in Sharp and Fuzzy settings, respectively

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- In practice, remember the following tips:
  - Do not use high-order polynomials as smoothing functions
  - A common way is to use local linear regression
  - Using packages in Stata to give you optimal bandwidth and bias-corrected inference
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