Frontier Topics in Empirical Economics: Week 6
IV beyond LATE

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IV beyond LATE: Limitation of LATE

- We have introduced LATE interpretation of IV
- This is the most popular way to think of IV under heterogeneous treatment effect
- It is elegant, policy-relevant, but also limited (Heckman and Vytlacil, 2007a,b)
  - It relies on binary treatment and binary IV
  - It is internally valid, but not externally valid
- Complier group is policy-specific, environment-specific
- When environment changes, complier group changes

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- First, we relax the assumption of binary treatment, single and binary IV
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- Second, we introduce a more general framework with better external validity: Marginal Treatment Effect (MTE)
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IV beyond LATE: Choice Model and IV

- Choice model is intrinsically nested in IV
- When you consider always-taker, complier, never-taker
- You are thinking about these people's choices under different policy shocks
- This choice structure is not fully utilized in pure design-based approach
- It can definitely help you when data is not enough to identify the effect
- The whole point of this lecture is to discuss how to use choice model and economic theory to regularize IV
- An interaction between design-based approach and structural approach

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- The idea of monotonicity comes from assuming treatment is a normal good
- If the agent chooses something when the price is higher ($D(z = 0) = 1$)
- Then he/she will definitely choose it when the price is lower ($D(z = 1) = 1$)

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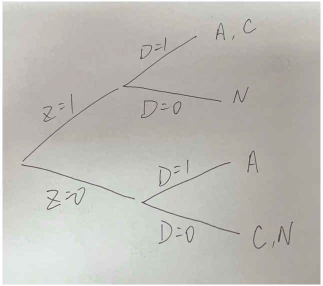
Generalization of LATE: Multiple IV

- In LATE theorem, we assume that both IV and treatment are single and binary
- Then it gives you $2 \times 2 = 4$ types of people (A,C,N,D)
- By assuming monotonicity, we eliminate D



- We have four equations (final nodes)
- LATE can be inverted from expectation functions from the four final nodes
- It can be identified by the IV estimator

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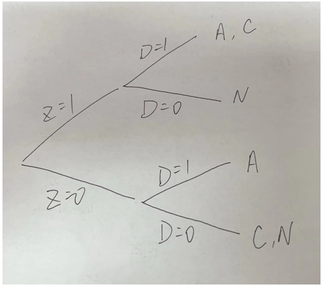

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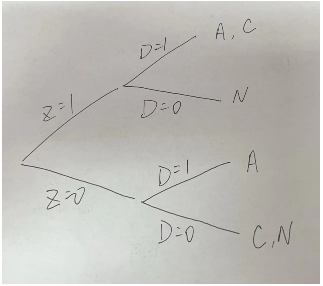

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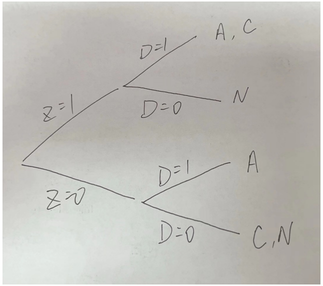

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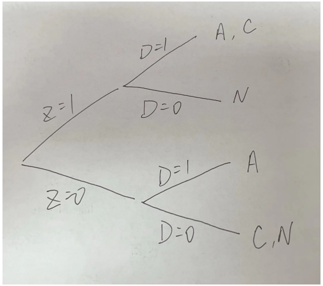

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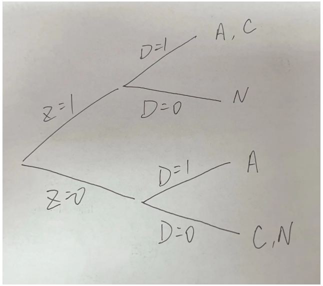

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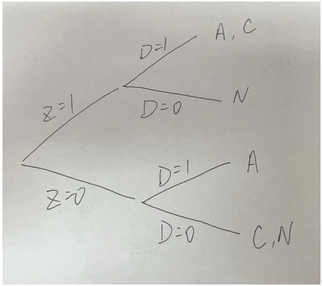

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- First, consider we have multiple binary IV and binary treatment
- This is relatively simple
- We run regressions taking z_1, z_2 as instruments (not z)
- Assuming monotonicity for both z_1 and z_2
- The corresponding IV estimator can be derived as:

$$\rho_{2SLS} = \psi LATE_1 + (1 - \psi) LATE_2$$

- $LATE_1, LATE_2$ are LATEs for instrument z_1 and z_2

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Generalization of LATE: Multivalued Treatment

- Now we consider multivalued treatment and binary IV: Average Causal Response (ACR)
- Assume that we have treatment $s \in \{0, 1, 2, \ldots, \bar{s}\}$
- For example, IV is the implementation of a compulsory education law
- Treatment is the education level, which takes multiple values
- We have the following three assumptions:
  - ACR1 Independence: $\{Y_{0i}, Y_{1i}, \ldots, Y_{\bar{s}i}, s_{0i}, s_{1i}\} \perp Z_i$
  - ACR2 First stage existence: $E[s_{1i} - s_{0i}] \neq 0$
  - ACR3 Monotonicity: $s_{1i} - s_{0i} \leq 0 \forall i$ or vice versa
- ACR3 implicitly requires us to have an "ordered" list of values for treatment

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- Under ACR1-3, IV identifies a weighted average of the unit causal response

Theorem 4.5.3 in MHE

When ACR1, ACR2, and ACR3 hold, we have

$$\frac{E[Y_i|z_i=1]-E[Y_i|z_i=0]}{E[s_i|z_i=1]-E[s_i|z_i=0]}=\sum_{s=1}^{\bar{s}}\omega_s E[Y_{si}-Y_{s-1,i}|s_{1i}\geq s>s_{0i}],$$

$$\text{where } \omega_s=\frac{P[s_{1i}\geq s>s_{0i}]}{\sum_{j=1}^{\bar{s}}P[s_{1i}\geq j>s_{0i}]}$$

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$$\frac{E[Y_i|z_i = 1] - E[Y_i|z_i = 0]}{E[s_i|z_i = 1] - E[s_i|z_i = 0]} = \sum_{s=1}^{\hat{s}} \omega_s E[Y_{si} - Y_{s-1,i}|s_{1i} \geq s > s_{0i}],$$

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Generalization of LATE: Multivalued Treatment

- Under ACR1-3, IV identifies a weighted average of the unit causal response

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Generalization of LATE: Multivalued Treatment

- $Y_{si} - Y_{s-1,i}$ is the unit response, or stepwise treatment effect
- For each unit/step change, we average over all compliers that cover this unit/step
- For instance, the unit change from $s = 1$ to $s = 2$ includes compliers
  - who choose $s = 0$ when $z = 0$, but choose $s = 2, 3, ..., S$ when $z = 1$
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- We have already checked multiple IV and multivalued treatment cases
- We now consider multivalued IV
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- We can first decompose the multivalued IV to multiple dummies
- Each dummy represents a specific value of IV
- For example, if $z = 0, 1, 2$, we have dummies z_1, z_2 as indicators
- $z_1 = 1$ if $z = 1$; $z_1 = 0$ if $z = 0, 2$
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- Then we run the regression using the set of dummies z_1, z_2 as instruments
- We can interpret the results as in multiple IV case
- But is this correct?
- An important assumption is monotonicity for each dummy IV
- However, it is not true for z_1
- Because for the group of people with $z_1 = 1$
- They can be either $z = 0$ or $z = 2$
- It is possible that $D_i(z_i = 0) < D_i(z_i = 1) < D_i(z_i = 2)$
- Then, for $z_1 = 1$, some people go to one direction ($z = 2$), some people go to the other, violating the monotonicity assumption

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- Thus, monotonicity assumption is not as innocuous as in the 2×2 case
- We need to go to deep choice structure of this assumption: Axiom of Revealed Preference
- In this case, you have to analyze one by one based on your specific context

WARP: Definition 2.F.1 MWG

The Walrasian demand function $x(p,w)$ satisfies the weak axiom of revealed preference if the following holds for any two price-wealth situations (p,w) and (p',w'):

$$\text{If } p \cdot x(p',w') \leq w \text{ and } x(p',w') \neq x(p,w), \text{ then } p' \cdot x(p,w) > w'$$

- If some optimal bundle in situation B is also feasible but not chosen in situation A, then the optimal bundle in situation A is not feasible in situation B ($x_A \succsim_R x_B$)
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- A stronger version of WARP is SARP

SARP Definition

The market demand function $x(p,w)$ satisfies the strong axiom of revealed preference if for any list of $(p^1,w^1),\ldots,(p^N,w^N)$

with $x(p^{n+1},w^{n+1}) \neq x(p^n,w^n)$ for all $n \leq N-1$

we have $p^N \cdot x(p^1,w^1) > w^N$ whenever $p^n \cdot x(p^{n+1},w^{n+1}) \leq w^n$ for all $n \leq N-1$

- SARP adds transitivity to WARP
- If $x_N \succsim_R x_{N-1}, x_{N-1} \succsim_R x_{N-2} \ldots x_2 \succsim_R x_1$, we have $x_N \succsim_R x_1$
- Let's go to the example of MTO in Pinto (2015)

Generalization of LATE: Multivalued IV

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The market demand function $x(p, w)$ satisfies *the strong axiom of revealed preference* if for any list of $(p^1, w^1), \ldots (p^N, w^N)$

with $x(p^{n+1}, w^{n+1}) \neq x(p^n, w^n)$ for all $n \leq N - 1$,

we have $p^N \cdot x(p^1, w^1) > w^N$, whenever $p^n \cdot x(p^{n+1}, w^{n+1}) \leq w^n$ for all $n \leq N - 1$

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- If $x_N \succsim_R x_{N-1}, x_{N-1} \succsim_R x_{N-2} \ldots x_2 \succsim_R x_1$, we have $x_N \succsim_R x_1$
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Generalization of LATE: Multivalued IV

- Moving to Opportunity (MTO) is a housing experiment to encourage low-income families to move to neighborhood with low poverty rate
- There are three policy groups (three values of IV)
  - Control group: No vouchers (z_1)
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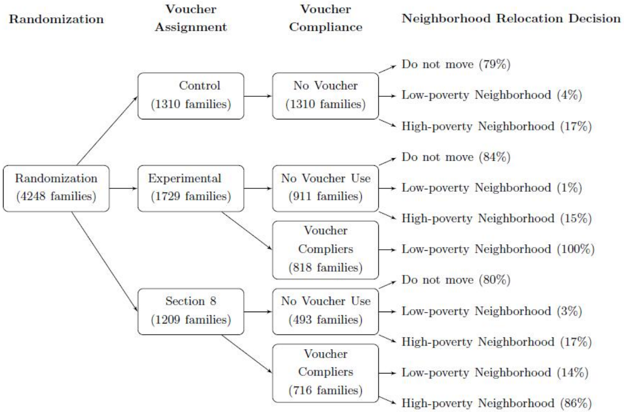
Figure 1: **Neighborhood Relocation by Voucher Assignment and Compliance**



Generalization of LATE: Multivalued IV

- Thus, we have $3 \times 3 \times 3 = 27$ types of agents
- Only 12 available equations for observed expectations
- It is impossible to invert a linear system of 9 equations to identify any causal effect with 27 behavior types
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- Let $u_\omega(k, t)$ be the utility function of family ω (k consumption, t relocation choice)
- Let $W_\omega(z, t)$ be the budget set of family ω under relocation decision $t \in \{1, 2, 3\}$ and MTO voucher $z \in \{z_1, z_2, z_3\}$
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- Now we translate three subsidizing rules to budget set:
  - Control group ($z_1 = 1$) subsidies nothing
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Assumption A-1, A-2 Pinto (2015)

According to the features of MTO, we assume the budget sets satisfy:

$$W_\omega(z_1, 2) \leq W_\omega(z_2, 2) = W_\omega(z_3, 2) \quad (1)$$

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- (1): If you choose to relocate to low poverty neighborhood ($t = 2$), your consumption would be higher if you are in Experimental or Section 8 groups
- (2): If you choose to relocate to high poverty neighborhood ($t = 3$), your consumption would be higher if you are in Section 8 group
- (3): If you choose not to relocate, or relocate to places that is not supported by your MTO group, your budget will not change

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Generalization of LATE: Multivalued IV

- Then we derive the following choice rule

Lemma L-1 Pinto (2015)

If preferences are rational, under Assumption A-1 and A-2

$$1. C_\omega(z_1) = 2 \Rightarrow C_\omega(z_2) = 2, C_\omega(z_3) \neq 1$$
$$2. C_\omega(z_1) = 3 \Rightarrow C_\omega(z_2) \neq 1, C_\omega(z_3) \neq 1$$
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- Test yourself, explain all these six inequalities

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Generalization of LATE: Multivalued IV

- We further assume that neighborhood is a normal good

Assumption A-3, Pinto (2015)

For each family ω, and for $z, z' \in \{z_1, z_2, z_3\}$, if $C_\omega(z) = r$ and $W_\omega(z, r)$ is a proper subset of $W_\omega(z', r)$, then $C_\omega(z') = r$

- To eliminate cases like $C_\omega(z_1) = 2, C_\omega(z_2) = 2, C_\omega(z_3) = 3$
- Using all above, we can eliminate the number of types from 27 to 7
- Now you see the power of economic theory to guide your identification
- When statistics tools are exhausted, remember you are an economist
- Do not think first year Micro and Macro are useless!!!

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For each family ω, and for $z, z' \in \{z_1, z_2, z_3\}$, if $C_\omega(z) = t$ and $W_\omega(z, t)$ is a proper subset of $W_\omega(z', t)$, then $C_\omega(z') = t$

- To eliminate cases like $C_\omega(z_1) = 2, C_\omega(z_2) = 2, C_\omega(z_3) = 3$
- Using all above, we can eliminate the number of types from 27 to 7
- Now you see the power of economic theory to guide your identification
- When statistics tools are exhausted, remember you are an economist
- Do not think first year Micro and Macro are useless!!!

Generalization of LATE: Multivalued IV

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MTE: Choice Model

- Now we go to the second part, how to improve the external validity
- The reason why LATE is lack of external validity is because it is defined on a policy-specific ex post group
- Not some ex ante group, for example a group of high-skilled workers
- Grouping by post-determined behavior, but not pre-determined characteristics
- This ex post group will change when policy environment changes

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- Now let's explicitly construct a model for agents' compliance behavior
- In this model, we suppress subscript for individuals
- Let $j = 0, 1$ be the treatment, Y_1, Y_0 be the potential outcomes

$$Y_1 = \mu_1(X, U_1) \tag{4}$$
$$Y_0 = \mu_0(X, U_0) \tag{5}$$

- X is a set of control variables, U is unobserved factor on outcome

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- Let D denote the choice of treatment, determined by a latent index model

$$D^* = \mu_D(Z) - V, \quad D = 1 \text{ if } D^* \geq 0; \;\; D = 0 \text{ otherwise} \tag{6}$$

- Z is an instrument that can change individual's choices, V is an unobserved factor
- For instance, Y is wage, D is college enrollment, Z is a policy to subsidize students from poor regions
- Agents observe everything. Econometricians observe (Z, X), but not (U_0, U_1, V)
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MTE: Choice Model

- We invoke five assumptions for this model
- (A-1) (U_0, U_1, V) are independent of Z conditional on X
  Independence
- (A-2) $\mu_D(Z)$ is nondegenerate conditional on X
  Z contain at least one element not in X
- (A-1) and (A-2) assure the existence of the instrument
- (A-3) The distribution of V is continuous
- (A-4) $E(|Y_1|), E(|Y_0|)$ are finite
- (A-5) $0 < Pr(D = 1|X) < 1$
  Possible to have $D = 1$ or $D = 0$ at any point of X

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MTE: Choice Model

- An example of this model setting is the Roy Model (sorting model)
- We have two sectors 0 and 1
- Y is working payoff, there is relative working cost $C = Z_1 + V_C$ in sector 1, Z_1 is observed and V_C is unobserved
- Agents choose a sector with higher payoff (abstract from cost)
- The unobserved term in treatment function is positively correlated with unobserved treatment return $\Longrightarrow$ Positive sorting
- People with higher return sort into treatment

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- Assume μ additively separable in U

$$Y_1 = \mu_1(X) + U_1$$
$$Y_0 = \mu_0(X) + U_0$$
$$D^* = \mu_1(X) + U_1 - [\mu_0(X) + U_0] - Z_1 - V_C, \quad D = 1 \text{ if } D^* \geq 0; \ D = 0 \text{ otherwise}$$

- In this case, we have $V = -[U_1 - U_0 - V_C]$
- Positive sorting: $Cov(U_1 - U_0, U_1 - U_0 - V_C) > 0$

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- Let $P(Z|X) \equiv Pr(D = 1|Z, X) = F_{V|X}(\mu_D(Z))$
  $F_{V|X}(\cdot)$ denotes the distribution of V conditional on X
- This is the propensity score to get treated for agent with Z
- Let $U_D = F_{V|X}(V)$, we have $U_D \sim Unif[0, 1]$
- $F_{V|X}(V)$ means the threshold propensity score the agent has to pass to get treated when he/she draws V
- Agent has to have an instrument Z which give him/her a propensity score $F_{V|X}(\mu_D(Z)) > F_{V|X}(V) = U_D$ (larger than this threshold) to get treated
- We have a clear one-to-one mapping between V and U_D
- Thus, for a choice function, an agent can be characterized by (X, V) or (X, U_D)

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- $F_{V|X}(V)$ means the threshold propensity score the agent has to pass to get treated when he/she draws V
- Agent has to have an instrument Z which give him/her a propensity score $F_{V|X}(\mu_D(Z)) > F_{V|X}(V) = U_D$ (larger than this threshold) to get treated
- We have a clear one-to-one mapping between V and U_D
- Thus, for a choice function, an agent can be characterized by (X, V) or (X, U_D)

MTE: Choice Model

- Let $P(Z|X) \equiv Pr(D = 1|Z, X) = F_{V|X}(\mu_D(Z))$
  $F_{V|X}(\cdot)$ denotes the distribution of V conditional on X
- This is the propensity score to get treated for agent with Z
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- Vytlacil (2002) proves that (A-1) to (A-5) in this additively separable selection model is equivalent to the LATE model of Imbens and Angrist (1994)
- The intuition is simple: V could not affect $\mu_D(Z)$
- $D^* = \mu_D(Z) - V \Rightarrow$ additively separable for Z and V
- Thus, given z and z', $\forall V \Rightarrow D^*(z) \geq D^*(z')$ or $D^*(z) \leq D^*(z')$
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MTE: Defining MTE

- Now let's define ATE and MTE in this model
- Let $\Delta = Y_1 - Y_0$
- ATE is defined as usual: $\Delta^{ATE}(x) \equiv E(\Delta | X = x)$
- MTE is defined as the mean effect of treatment on those for whom $X = x$ and $U_D = u_D (V = v)$

Defining the MTE

The Marginal Treatment Effect is defined as:

$$\Delta^{MTE}(x, u_D) \equiv E(\Delta | X = x, U_D = u_D)$$

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- People with observed characteristics X and unobserved taste on treatment V
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- That is why it is called "marginal"
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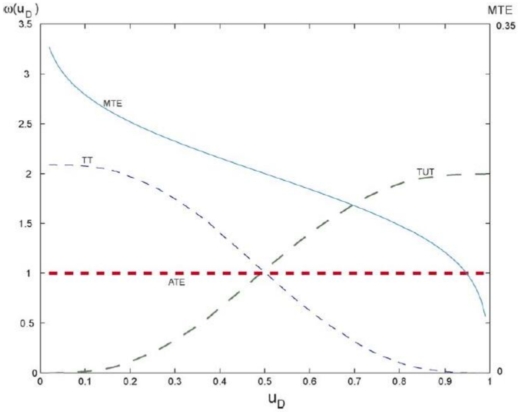
MTE: Defining MTE

- Selection on MTE in a positive sorting Roy model



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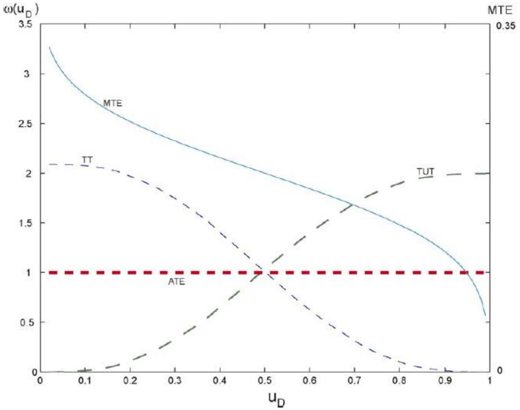

MTE: MTE as a Framework

- We can prove that MTE is a general framework with various causal parameters as its special cases
- LATE can be written as a weighted average of MTE:

$$
\begin{aligned}
LATE &= E(Y_1 - Y_0 | X = x, D(z) = 1, D(z^{'}) = 0) \\
&= E(Y_1 - Y_0 | X = x, u_D^{'} < U_D \leq u_D) \\
&= \int_{u_D^{'}}^{u_D} \Delta^{MTE}(x, u) du
\end{aligned}
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- Here $u_D = Pr(D(z) = 1), u_D^{'} = Pr(D(z^{'}) = 1)$ are the threshold propensity scores for instrument $Z = z$ and $Z = z^{'}$
- We can interpret LATE as the average TE for people whose threshold is below z but above $z^{'}$

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MTE: MTE as a Framework

- In general, we can express treatment parameter j by MTE as:

$$TE(j) = \int_0^1 \Delta^{MTE}(x, u_D)\omega_j(x, u_D)du_D$$

- ω_j is the weight for j

$\text{ATE}(x) = E(Y_1 - Y_0 \mid X = x) = \int_0^1 \Delta^{\text{MTE}}(x, u_D)\, du_D$

$\text{TT}(x) = E(Y_1 - Y_0 \mid X = x, D = 1) = \int_0^1 \Delta^{\text{MTE}}(x, u_D)\omega_{\text{TT}}(x, u_D)\, du_D$

$\text{TUT}(x) = E(Y_1 - Y_0 \mid X = x, D = 0) = \int_0^1 \Delta^{\text{MTE}}(x, u_D)\omega_{\text{TUT}}(x, u_D)\, du_D$

Policy relevant treatment effect: $\text{PRTE}(x) = E(Y_{a'} \mid X = x) - E(Y_a \mid X = x) = \int_0^1 \Delta^{\text{MTE}}(x, u_D)\omega_{\text{PRTE}}(x, u_D)\, du_D$ for two policies a and a' that affect the Z but not the X

$\text{IV}_J(x) = \int_0^1 \Delta^{\text{MTE}}(x, u_D)\omega_{\text{IV}}^J(x, u_D)\, du_D$, given instrument J

$\text{OLS}(x) = \int_0^1 \Delta^{\text{MTE}}(x, u_D)\omega_{\text{OLS}}(x, u_D)\, du_D$

Source: Heckman and Vytlacil (2005).

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MTE: Estimate MTE Using LIV

- Now we have defined MTE and shown that it is a general framework
- We suppress notation of conditional on x
- How to identify it? Local instrumental variable (LIV)
- LIV is the derivative of the conditional expection of Y w.r.t $P(Z) = p$:

$$\Delta^{LIV}(p) \equiv \frac{\partial E(Y|P(Z) = p)}{\partial p}$$

- LIV is the mean response to a policy change embodied in changes in $P(Z)$
- A policy shock changes $Z \Longrightarrow$ changes propensity score $P(Z) \Longrightarrow$ changes outcome Y

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- For MTE at any propensity threshold p, we can use LIV at this point to identify it
- What is the intuition?
- MTE at a threshold means the causal effect on marginal people who would just change their treatment at this point of $P(z) = p$
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$$\Delta^{MTE}(p) = \Delta^{LIV}(p) = \frac{\partial E(Y|P(Z) = p)}{\partial p}$$

- For MTE at any propensity threshold p, we can use LIV at this point to identify it
- What is the intuition?
- MTE at a threshold means the causal effect on marginal people who would just change their treatment at this point of $P(z) = p$
- LIV is the changes of outcome at this marginal point $P(Z) = p$ driven by an exogenous variation on instrument Z

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- Then the question becomes how to estimate LIV?
- First, assume a treatment choice function (Probit or logit), find propensity score function $p(z)$
- Second, estimate outcome Y given control X and propensity score function $p(z)$
  Using non/semi-parametric methods such as local linear regression or partial linear regression
- Then estimate derivatives by small perturbation
  Or it would be just the regression coefficient if you assume a linear model for Y
- Or we can estimate the whole model in a fully parametric way (Kline and Walters, 2016)

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- Implementation: Stata package *mtefe*
- This package can give you estimations of various causal parameters
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```
. mtefe_gendata, obs(10000) districts(10)
.
. mtefe lwage exp exp2 i.district (col=distCol)
Parametric normal MTE model                        Observations : 10000
Treatment model: Probit
Estimation method: Local IV
```

lwage	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
beta0						
exp	.0358398	.0064408	5.56	0.000	.0232145	.0484651
exp2	-.0008453	.0002019	-4.19	0.000	-.0012411	-.0004496
district						
2	.2352456	.0680412	3.46	0.001	.1018712	.36862
3	.6294914	.0701091	8.98	0.000	.4920634	.7669194
4	.0131179	.0597721	0.22	0.826	-.1040474	.1302832
5	.0338606	.0705835	0.48	0.631	-.1044974	.1722186
6	.1699366	.0605086	2.81	0.005	.0513275	.2885458
7	-.1899241	.060115	-3.16	0.002	-.3077617	-.0720865
8	-.1842254	.0676843	-2.72	0.007	-.3169003	-.0515504
9	-.7908301	.0578436	-13.67	0.000	-.9042153	-.677445
10	-.4432749	.0597237	-7.42	0.000	-.5603455	-.3262044
_cons	3.164706	.0650331	48.66	0.000	3.037228	3.292184
beta1-beta0						

MTE: Estimate MTE Using LIV

exp	-.0386384	.010241	-3.77	0.000	-.0587128	-.018564
exp2	.0012967	.0003288	3.94	0.000	.0006523	.0019412
district						
2	.265112	.107039	2.48	0.013	.0552939	.4749301
(*output omitted*)						
10	.3143661	.1072555	2.93	0.003	.1041237	.5246085
_cons	.4255863	.0983572	4.33	0.000	.2327863	.6183863
k						
mills	-.4790282	.0611081	-7.84	0.000	-.5988124	-.359244
effects						
ate	.3283373	.0242932	13.52	0.000	.2807177	.3759568
att	.5369432	.0388809	13.81	0.000	.4607287	.6131576
atut	.1195067	.0384691	3.11	0.002	.0440995	.194914
late	.3279726	.0245142	13.38	0.000	.2799198	.3760254
mprte1	.3463148	.0256971	13.48	0.000	.2959433	.3966862
mprte2	.3309428	.024298	13.62	0.000	.2833137	.3785719
mprte3	-.016257	.0498984	-0.33	0.745	-.1140679	.0815538

```
Test of observable heterogeneity, p-value                    0.0000
Test of essential heterogeneity, p-value                     0.0000

Note: Analytical standard errors ignore the facts that the propensity score,
```
(*output omitted*)

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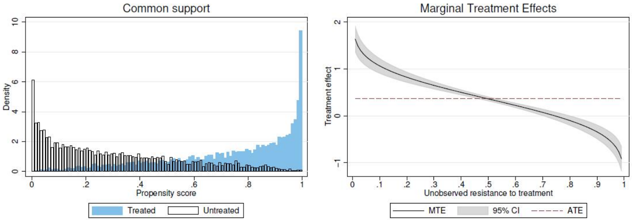

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- LATE is internally valid but not externally valid
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- MTE measures the treatment effect for people with specific characteristics X and some unobserved treatment taste V (or treatment threshold p)
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- We illustrate the method we learn today by reading Kline and Walters (2016)
- This paper is so interesting and insightful
- Reading one paper like this carefully, is much better than reading 100 reg monkey papers (for these, you can just read the abstracts)
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- People find large impact from observational studies, but small effect from RCT. Does it mean that this HS is ineffective?
- Kline and Walters (2016) claim that it is not because observational studies are not well-designed
- Rather, it is because observational studies compare people enroll in HS and people do not enroll in any program
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- The treatment actually has three values: no program, other program, HS program
- Kline and Walters (2016) first categorize people to all behavior types and use ARP to eliminate some of them
- Then they varify various causal parameters needed for different evaluation targets
  - ITT and LATE: not externally valid when the composition of compliers changes
  - MTE: externally valid when the composition of compliers changes
- Using these causal estimates, they implement new cost-benefit analysis
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Conclusion

- LATE is the most popular way to interpret IV estimate
- However, it has two important limitations
  - Usually not feasible when you have multivalued IV $\implies$ too many types
  - Not externally valid when complier group changes
- To fix these two issues, we need to go deep into the compliance (treatment selection) problem
- Treatment selection is intrinsically a part of IV, but not fully explored by pure design-based approach

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- First, we use ARP and other reasonable economic assumptions to simplify the identification in complicated multivalued IV cases
- Second, we introduce MTE framework to deal with external validity issues
- MTE is the treatment effect of a small group of people with specific value of characteristics X and treatment taste V (or treatment threshold U_D)
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