Frontier Topics in Empirical Economics: Week 1 Outline of Causal Inference

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November 30, 2023

Plan of This Course

- Basic causal inference and model selection (Week 1-4)
  Potential Outcome Framework, RCT, matching vs regression, non-parametric method, machine learning, DAG framework
- IV (Week 5-7)
  IV, LATE, GMM, MTE, Bartik IV
- Causal inference with panel data (Week 8-9)
  Basic DID and event study, pre-trend testing, synthetic control, staggered DID
- Other Topics (Week 10-11)
  RDD, Std err issues
- Introduction to discrete choice model (Week 12-13)
  Logit, Probit, Nested Logit, Control function, BLP

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- The goal of this course is to let all students avoid being regression monkeys
- What is regression monkey? $\Longrightarrow$ Run regs without creativity
  - Running regressions without knowing why
  - Only know very basic statistical off-the-shelf methods
  - Have no economic sense, do not know any economic theory
- This is no economist, this is BAD statistician!
- This course aims to teach you
  - The logic behind regression and causal inference
  - Statistical tools beyond regression in causal inference
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Motivating Example: Female Labor Participation

This is an example from Professor Chao Fu.

- Consider a female labor participation problem
- Utility maximization of female i:

$$\begin{aligned} max \quad & U_i(c_i, 1 - l_i) + \epsilon_{il} \\ s.t. \quad & c_i = w_i l_i \end{aligned} \tag{1}$$

c_i: consumption; l_i: labor supply; ϵ_{il}: unobserved taste shock; w_i: wage

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Motivating Example: Female Labor Participation

- Assume that l_i is binary (work, not work)
- $l_i = 1$ if $U(l = 1) \geq U(l = 0)$:

$$U_i(w_i, 0) + \epsilon_{i1} \geq U_i(0, 1) + \epsilon_{i0} \tag{2}$$

- Then given w_i, we have a threshold value of $\epsilon_{i1} - \epsilon_{i0}$ for i to choose to work:

$$l_i = 1 \quad \text{if} \quad \epsilon_{i0} - \epsilon_{i1} < \epsilon^* \tag{3}$$
$$\epsilon^* = U_i(w_i, 0) - U_i(0, 1)$$

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- Assume that shock $\epsilon_{i1} - \epsilon_{i0}$ has a CDF $F_{\epsilon|w,chi}$
- We have the following working probability for i:

$$G(w, chi) = Pr(I = 1|w) = \int_{-\infty}^{\epsilon^*} dF_{\epsilon|w}$$
$$= F_{\epsilon|w}(\epsilon^*(w)) \quad (4)$$

- Two empirical research approaches for this question

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1. We can directly estimate probability function G
   - Assume that G is a linear function

$$G(w) = \beta_0 + \beta_1 w_i \tag{5}$$

   - Linear Probability Model $\Longrightarrow$ We can use OLS to estimate β
   - This is called "Reduced-form" approach
   - We usually identify it by some research "design" (IV, RDD, DID)
   - Thus, it is also called "Design-based" approach

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Motivating Example: Female Labor Participation

2. We can estimate ϵ's CDF F, and utility function U

- We have the likelihood function as:

$$L(\Theta^U, \Theta^F; data) = \prod_{i=1}^{N} F_\epsilon(\epsilon^*)^{l_i}[1 - F_\epsilon(\epsilon^*)]^{1-l_i} \quad (6)$$

  Θ^U is the parameter set of utility function; Θ^F is the parameter set of shock's CDF

- We use MLE to estimate Θ^U and Θ^F $\Longrightarrow$ Recover choice structure directly
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Motivating Example: Female Labor Participation

For example,

- $U = \alpha w_i + \phi(1 - l_i)$
- ϵ follows T1EV distribution
- We have the likelihood function as:

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\begin{aligned}
L(\Theta^U, \Theta^F; data) &= \prod_{i=1}^{N} F_\epsilon(\epsilon^*)^{l_i}[1 - F_\epsilon(\epsilon^*)]^{1-l_i} \\
&= \prod_{i=1}^{N} \left(\frac{exp(\alpha w_i)}{exp(\alpha w_i) + exp(\phi)}\right)^{l_i} \times \left(\frac{exp(\phi)}{exp(\alpha w_i) + exp(\phi)}\right)^{1-l_i} \qquad (7)
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Two Approaches: Internal vs External Validity

- Before we compare the two approaches
- We need to first clarify what does it mean by "internal" and "external validity
- Internal means that it is valid within some current specific context or environment
- External means that it is also valid outside the current context or environment
- External refers to our attempt to extrapolate our analysis

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- Take One Child Policy (OCP) as an example
- There are three layers of policy evaluation (Heckman and Vytlacil, 2007)
  - Evaluating the impact of a historical intervention
    What was the impact of the OCP on fertility rate?
  - Forecasting the impact of an intervention previously happened in environment A to happen in another environment B
    What would be the impact if we restart the OCP in 2023?
  - Forecasting the impact of an intervention never happened in history in any environment
    What would be the impact if we force all women to give birth to at least one child?
- The first one is "internal"
- The second and the third one are "external"

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    What would be the impact if we force all women to give birth to at least one child?
- The first one is "internal"
- The second and the third one are "external"

Two Approaches: Internal vs External Validity

- Take One Child Policy (OCP) as an example
- There are three layers of policy evaluation (Heckman and Vytlacil, 2007)
  - Evaluating the impact of a historical intervention
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- Target: Primitive parameters $\Longrightarrow$ Choice structure
  Agent's utility function, firm's production function, market structure...
- Advantages
  - Deeper economic thinking: we can understand the original decision-making process
  - Great external validity $\Longrightarrow$ Solid under Lucas' critique
  - More reliable counterfactual analysis
- Disadvantages
  - Need more (untestable) assumptions on functional form, distribution of unobservable...
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Two Approaches: Reduced-form/Design-based Approach

- Target: Some marginal effect of conditional expectation function
  What is the impact of A on B?
- Do not care about the mechanism $\Longrightarrow$ A black box of causal effect
- Advantages
  - Very reliable if you have a good exogenous shock
  - Great internal validity, not so many assumptions
- Disadvantages
  - No mechanism is revealed $\Rightarrow$ More of a statistician than an economist
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- This course will mainly focus on the Reduced-form/Design-based Approach
- Specifically, I will carefully go through traditional regression tools used in Applied Econ
- And introduce tools beyond simple regression
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- In general, let's try not to be Reg Monkeys!

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- Example : Health status and hospitalization

Group	Sample Size	Mean Health Status
Hospital	7,774	3.21
No hospital	90,049	3.93

- Going to hospital makes you more sick?
- No! People go to hospital because they are sick.
- Correlation is NOT causality!!!

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- Binary treatment D_i for individual i, some outcome Y_i
- Y_{0i}: The "potential outcome" of i if he/she is not treated, regardless of the treatment status in reality
- Y_{1i}: The "potential outcome" of i if he/she is treated, regardless of the treatment status in reality
- Thus, we have:

$$Y_i = \begin{cases} Y_{1i} & \text{if} \quad D_i = 1 \\ Y_{0i} & \text{if} \quad D_i = 0 \end{cases} \tag{8}$$
$$= Y_{0i} + (Y_{1i} - Y_{0i})D_i$$

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- Individual treatment effect: $Y_{1i} - Y_{0i}$
- Not available: There is only one world! Given i, you see either Y_{0i} or Y_{1i}
- But we can consider averages: By differencing mean outcomes from the two groups

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\begin{aligned}
& E[Y_i|D_i = 1] - E[Y_i|D_i = 0] \qquad (9) \\
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Potential Outcome Framework and RCT

Randomization can solve the selection problem.

- Assume that we randomly assign the treatment to the population:

$$D_i \perp\!\!\!\perp Y_{0i}, Y_{1i} \tag{10}$$

- Then we have selection bias to be zero:

$$E[Y_{0i}|D_i = 1] - E[Y_{0i}|D_i = 0] = 0$$

- Thus, simple difference between the mean of treated and untreated group is ATT (and overall ATE)

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$$E[Y_i|D_i = 1] - E[Y_i|D_i = 0] = E[Y_{1i}|D_i = 1] - E[Y_{0i}|D_i = 1] = ATT = ATE$$

Regression, CEF and Causal Inference

- Regression is the most useful tool in applied econometrics
- When can we interpret regression coefficient as causal effect?
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Regression, CEF and Causal Inference

Conditional Expectation Function (CEF)

- CEF is the conditional expectation of an outcome Y_i, given some predictor vector X_i

$$E[Y_i|X_i = x] = \int tf_y(t|X_i = x)dt \tag{11}$$

where f_y is pdf
- This is a population concept ($n \to \infty$)
- It describes a prediction of X on Y, but NOT necessarily causal
- We can always decompose Y_i as predicted part (CEF) + error part

$$Y_i = E[Y_i|X_i] + \epsilon_i \tag{12}$$

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Regression, CEF and Causal Inference

- CEF is the best predictor of Y_i given X_i
- It minimizes the mean squared prediction errors

Theorem: CEF and MMSE

Let $m(X_i)$ be any function of X_i. The CEF solves

$$E[Y_i|X_i] = \operatorname{argmin}_{m(X_i)} E[(Y_i - m(X_i))^2]$$

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Regression, CEF and Causal Inference

Linear Regression

- Regression is a linear prediction that minimizes the mean squared error

$$Y_i = X_i'\beta + \epsilon_i$$

$$\beta = argmin_b E[(Y_i - X_i'b)^2]$$

- We have the first order condition (moment condition) as:

$$E[X_i(Y_i - X_i'\beta)] = 0$$

- The solution can be written as:

$$\beta = E[X_i X_i']^{-1} E[X_i Y_i]$$

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Regression, CEF and Causal Inference

Tips: Difference between β and $\hat{\beta}_{OLS}$

- Definition

$$\beta = E[X_i X_i']^{-1} E[X_i Y_i]$$

$$\hat{\beta}_{OLS} = (X'X)^{-1} X'Y$$

- $\hat{\beta}_{OLS}$ is an estimator of β (there can be alternative estimators, e.g. MLE)
- Population vs Sample, Identification vs Estimation
- X_i is an $1 \times k$ vector, Y_i is a scalar. They are random variables
- X is an $n \times k$ matrix, Y is an $n \times 1$ vector. They are realizations of random variables (real data)

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Regression, CEF and Causal Inference

CEF and linear regression

- $E[\epsilon_i|X_i] = 0$ vs $E[X_i\epsilon_i] = 0$
- Minimizing MMSE: Best predictor (CEF) vs Best linear predictor (linear regression)
- CEF is stronger than linear regression
- If CEF is linear, then linear regression is identical to CEF
- Even if CEF is not linear, regression is the best linear approximation to CEF (Minimize MSE)

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When does a regression coefficient have a causal meaning?

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Regression, CEF and Causal Inference

Regression and Causality

Case 1: We assume randomization (no need for controls) and constant TE

- When we have a random experiment with $D_i \perp\!\!\!\perp Y_{0i}, Y_{1i}$ and regression

$$Y_i = \alpha + \rho D_i + \epsilon_i$$

- If CEF is linear, then:

$$\rho = E[Y_i|D_i = 1] - E[Y_i|D_i = 0] = E[Y_{1i} - Y_{0i}|D_i = 1]$$

- Regression coefficient ρ is the ATT/TE

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Case 2: We assume randomization after controls

- Key to go from correlation/prediction to causality: Conditional Independent Assumption (CIA)/Selection on Observables

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Regression and Causality
Case 2: We assume randomization after controls

- Homogeneous (constant) treatment effect case is simple
- Assume linear CEF, for each $X_i = x$, we have the following regression:

$$Y_i = \alpha + \rho_r D_i + X_i' \gamma + \nu_i \tag{13}$$

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- Heterogeneous treatment effect case is more complicated
- Let δ_x be the within group ATE:

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- It can be shown that ρ_r is the treatment-variance weighted average of δ_x:

$$\rho_r = \frac{E[\sigma_D^2(X_i)\delta_x]}{E[\sigma_D^2(X_i)]} \tag{14}$$

- Proof see MHE Chapter 3.3.1
- Important! How to understand/interpret equation (14)? Give me an example

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$$\rho_r = \frac{E[\sigma_D^2(X_i)\delta_x]}{E[\sigma_D^2(X_i)]} \tag{14}$$

- Proof see MHE Chapter 3.3.1
- Important! How to understand/interpret equation (14)? Give me an example

Regression, CEF and Causal Inference

Regression and Causality

Case 2: We assume randomization after controls

- Heterogeneous treatment effect case is more complicated
- Let δ_x be the within group ATE:

$$\delta_x = E[Y_i|X_i, Di = 1] - E[Y_i|X_i, Di = 0] = E[Y_{1i}|X_i, Di = 1] - E[Y_{0i}|X_i, D_i = 1]$$

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Regression, CEF and Causal Inference

Let's compare assumptions of Regression, CEF and Causal Model

- $y = f(D) + e$
- Linear Regression: $f(D) = \beta D, E(De) = 0$ Uncorrelated
- CEF: $E(e|D) = 0$ Mean Independence
- Causal Model: $e \perp\!\!\!\perp D \quad (D_i \perp\!\!\!\perp y_{0i}, y_{1i})$ Independence
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Regression, CEF and Causal Inference

Main takeaways from this part

- Strength of assumptions regarding unobservable e
  Causal model (CIA) > CEF (Mean Independence) > Linear regression (Uncorrelated)
- CEF is the best predictor of Y given X
- Linear regression is the best linear predictor of Y given X
- Linear regression is the best linear approximation of CEF
- Under CIA and homogeneous TE, regression coefficient is the TE
- Under CIA and heterogeneous TE, regression coefficient is the treatment-variance weighted average of group ATE

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- We examine the effect of the treatments by patients' conditions (mild/severe)
- We have the death rate by treatments and conditions as:

	Mild	Severe	Total
A	15% (210/1400)	30% (30/100)	16% (240/1500)
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- Total death rate: A < B
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- C causes T and Y; T causes Y
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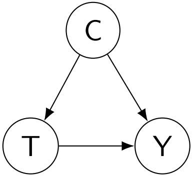

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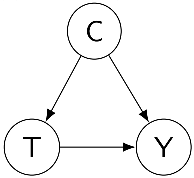

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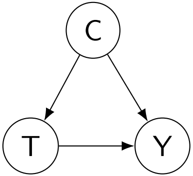

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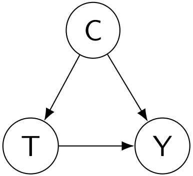

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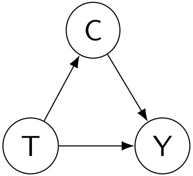

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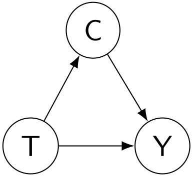

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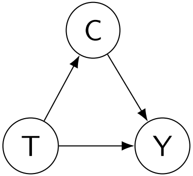

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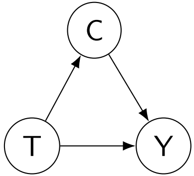

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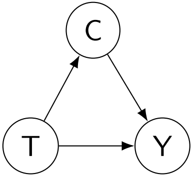

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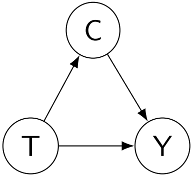

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Matching

- Regression is only one of the tools we use to tackle causal effect
- Matching is another common tool
- It is simple and non-parametric
- Basic idea
  - (1) Compare treated and control units with same covariates
  - (2) Put together to produce a single overall weighted average treatment effect
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- Assume that for treatment D_i, we have CIA: $Y_{0i}, Y_{1i} \perp\!\!\!\perp D_i | X_i$
- We can express treatment on the treated (TOT) as:

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\delta_{TOT} &= E[Y_{1i} - Y_{0i} | D_i = 1] = E[E[Y_{1i} - Y_{0i} | X_i, D_i = 1] | D_i = 1] \\
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- The corresponding matching estimator (sample analog) is:

$$\hat{\delta}_{TOT} = \sum_x \hat{\delta}_x \hat{P}(X_i = x | D_i = 1)$$

- Similarly, we can derive a matching estimator for ATE:

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- The corresponding matching estimator (sample analog) is:

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- Similarly, we can derive a matching estimator for ATE:

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- Assume that for treatment D_i, we have CIA: $Y_{0i}, Y_{1i} \perp\!\!\!\perp D_i | X_i$
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Regression is one of the matching estimators!

- Matching estimator of TOT: $\hat{\delta}_{TOT} = \sum_x \hat{\delta}_x P(X_i = x | D_i = 1)$
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Propensity Score Matching

- Assume that we want to estimate college premium on wages
- To have CIA, we need a lot of controls:
  Gender, race, nationality, birth weight, IQ, parents' education, parents' income...
- Curse of dimensionality: There are too many dimensions in X_i
- We will not have enough observations for each value of X_i to estimate $\hat{\delta}_x$
- Maybe you have 10,000 observations
- But only 2 of them are Han boys with IQ 150, family income 100,000 RMB/year, parents are high-school educated
- Very hard to implement the matching estimator (but regression is still feasible)

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- Assumption 1 (CIA): $Y_{1i}, Y_{0i} \perp\!\!\!\perp D_i | X_i$
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- Instead of matching across all controls (gender, family income...)
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- Because some parts of the process to implement PSM are not standardized
- e.g. how to estimate the propensity score $P(X)$? (Logit? LPM? Probit?)
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References

Heckman, James J and Edward J Vytlacil. 2007. "Econometric Evaluation of Social Programs, Part I: Causal Models, Structural Models and Econometric Policy Evaluation." *Handbook of Econometrics* 6:4779–4874.