Frontier Topics in Empirical Economics: Week 4
Directed Acyclic Graph

Zibin Huang [1]

[1] College of Business, Shanghai University of Finance and Economics

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Introduction

- Causal inference is the central topic of applied economics
- We almost solely focus on potential outcome framework in Economics
- This framework is proposed by Donald Rubin (Imbens and Rubin, 2015; Rubin, 1974) and sometimes called "Rubin Causal Model"

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- Is this the only statistical framework dealing with causal inference issue?
  Of course NOT.
- Graphical Model is another important method (Pearl, 2009)
- Today we are going to learn this new framework
- How it can be applied to economic research is still a very very open question
- Imbens wrote an interesting and critical paper on it
  Imbens (2020) Potential Outcome and Directed Acyclic Graph Approaches to Causality: Relevance for Empirical Practice in Economics

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- Introduce the graphical model and the DAG framework
- Discuss the possible usage of DAG for economists: Pros and Cons
- Compare DAG and PO framework: why PO is still more popular
- An example of using DAG: Pinto (2015)
- Conclusion: How can DAG help applied economics research (open question)

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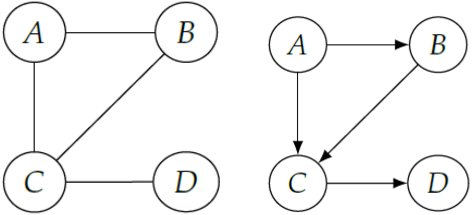
DAG Approach: Graph

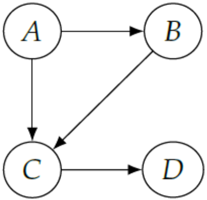
- Graph is a collection of *nodes* and *edges* that connect the nodes.
- Two nodes are called *adjacent* if they are connected by an edge.
- A directed graph's edges go out of a *parent* into a *child*.
- A *path* is any sequence of adjacent nodes, regardless of the direction of the edges. A *directed path* is a path that consists of directed edges that are all directed in the same direction.



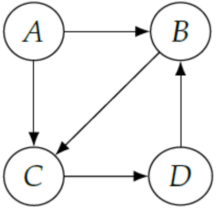
(a) Undirected Graph (b) Directed Graph

DAG Approach: Graph

- If there is a directed path that starts at node X and ends at node Y, then X is an ancestor of Y, and Y is a descendant of X.
- If there is no cycle in a directed graph, the graph is called a *directed acyclic graph* (DAG)



(c) Directed Graph



(d) Directed Graph with Cycle

DAG Approach: Bayesian Networks

- How to connect graphs to causal inference?
- The first step is to connect graphs to statistical relations: Bayesian Networks
- For any PDF, a Bayesian factorization can be expressed as:

$$P(x_1, x_2, ..., x_n) = P(x_1) \prod_{i \neq 1} P(x_i | x_{i-1}, ..., x_1) \quad (1)$$

- Example: $P(x_1, x_2, x_3) = P(x_1)P(x_2|x_1)P(x_3|x_2, x_1)$
- We can simplify the model if we assume some dependency structure, e.g. $P(x_3|x_2, x_1) = P(x_3|x_2)$ if $x_1 \perp x_3 | x_2$
- We can use a graph to represent this assumed dependency structure, system of probabilistic relations!
- A one-to-one mapping between graph G and probabilistic relations P

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DAG Approach: Bayesian Networks

Assumption (Minimality Assumption)

1. Given its parents in the DAG, a node X is independent of all its non-descendants (Local Markov Assumption);
2. Adjacent nodes in the DAG are dependent (Minimal independence).

Definition (Bayesian Network Factorization)

Given a probability distribution P and a DAG G satistying "Minimality Assumption", P factorizes according to G by

$$P(x_1, x_2, ..., x_n) = P(x_1) \prod_i P(x_i | pa_i)$$

where pa_i is the parents set of i.

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DAG Approach: Bayesian Networks

- Local Markov means that the dependence structure is "local" and "Markov"
- Minimal independence means that there is no more independence outside the graph
- Bayesian Factorization means that: If P has a causal structure as shown in G
  - X only depends on parents in the graph
  - We can do Bayesian network factorization for P w.r.t. G
- We call "G represents P", "G and P are compatible", "P is Markov relative to G"

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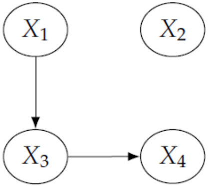
DAG Approach: Bayesian Networks

- Let's see a simple example
- Assume that we have four variables x_1, x_2, x_3, x_4
- A full decomposition is:

$$P(x_1, x_2, x_3, x_4) = P(x_1)P(x_4|x_3, x_2, x_1)P(x_3|x_2, x_1)P(x_2|x_1) \quad (2)$$

- What if we have the following DAG showing the relation among x_1, x_2, x_3, x_4?



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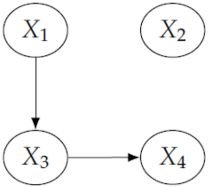

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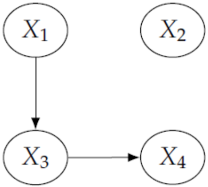

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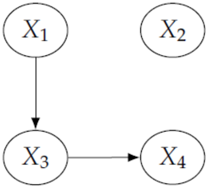

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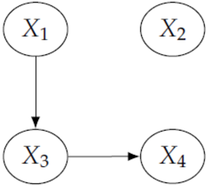

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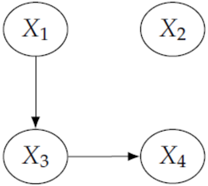
- Edges in the graph mean statistical dependencies!



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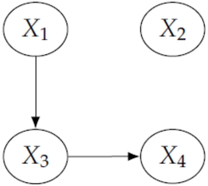

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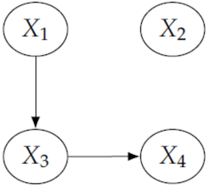

DAG Approach: Causal Graphs

- Up until now, we consider only statistical dependencies
- What about those arrows?

Assumption (Causal Edges Assumption)

In a directed graph, every parent is a direct cause of all its children

- By adding causal edge assumption, we have this DAG to represent not only statistical dependencies, but causal relations
- Directed paths in DAGs correspond to causation
- A more mathematically rigorous definition is imposed on SEM

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DAG Approach: Causal Graphs

- Up until now, we consider only statistical dependencies
- What about those arrows?

Assumption (Causal Edges Assumption)

In a directed graph, every parent is a direct cause of all its children.

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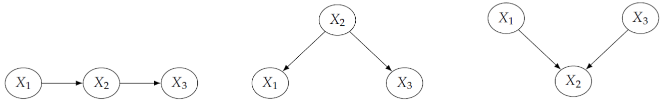
DAG Approach: Graphical Building Blocks

- Now we introduce some building blocks of the causal graph



(a) Chain (b) Fork (c) Immorality

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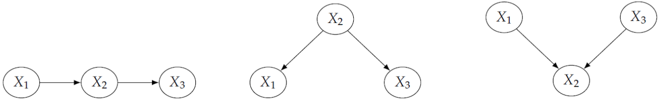

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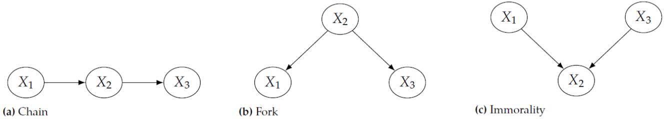

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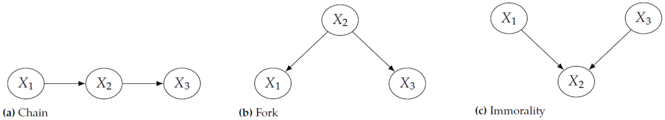

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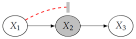

Figure 3.14: Chain with association blocked by conditioning on X_2.

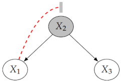

Figure 3.15: Fork with association blocked by conditioning on X_2.

- We can show that with this graph:

$$P(x_1, x_3 | x_2) = P(x_1 | x_2) P(x_3 | x_2) \tag{4}$$

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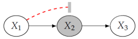

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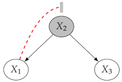

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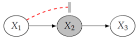

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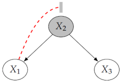

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- Things can be different in immorality
- We call X_2, the child of a immorality, as a *collider*

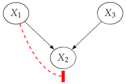

Figure 3.16: Immorality with association blocked by a collider.

- Applying Bayesian factorization:

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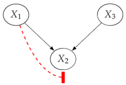

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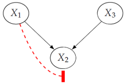

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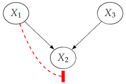

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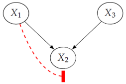

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- What's more, by conditional on x_2, you are creating dependencies!
- Controlling for post-determined variables!
- A simple example: x_1 is good-looking, x_2 is kindness, x_3 is marriage availability
- Conditional on $x_3 = 1$, you will see negative relation between x_1 and x_2!
- Well-known as bad control problem in econometrics

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DAG Approach: Blocked Path and d-separation

Definition (Blocked Path)

A path between X and Y is blocked by a conditioning set Z if either of the following is true:
1. Along the path, there is a chain $\rightarrow W \rightarrow$ or a fork $\leftarrow W \rightarrow$ where $W \in Z$;
2. There is a collider W that both itself and its descendants are not conditioned on in Z;

- Association flows along unblocked paths, does NOT flow along blocked paths!

Definition (d-separation)

Two sets of nodes X and Y are d-separated by a set of nodes Z if all of the paths between nodes in X and nodes in Y are blocked by Z

- d-separation means conditional independence!!
- All association flows between X and Y are blocked by Z

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- Theorem 1.2.4, 1.2.5 in Pearl (2009), Theorem 3.1 in Neal (2020)

Theorem (d-separation and statistical independence)

If X and Y are d-separated in a DAG G conditional on Z, then X and Y are independent conditioned on Z in every distribution compatible with G:

$$X \perp_G Y | Z \Rightarrow X \perp_P Y | Z \ \forall P \text{ compatible with } G$$

Conversely, if X and Y are independent conditional on Z in all P compatible with G, then X and Y are d-separated in G conditional on Z.

$$\forall P \text{ compatible with } G, X \perp_P Y | Z \Rightarrow X \perp_G Y | Z$$

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- Causations flow along directed unblocked paths
- Identification: how to net causation out of associations?
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- We define operator "$do(T = t)$" as an intervention to give the whole population treatment t
- We denote it in terms of potential outcomes as:

$$P(y|do(t)) = P(Y = y|do(T = t)) = P(Y(t) = y) \tag{6}$$

- $P(y|do(t))$ means the distribution of the potential outcome $P(Y(t) = y)$
- Identification of a causal model: If we can reduce an expression Q with *do* to one without *do*, then Q is identifiable.
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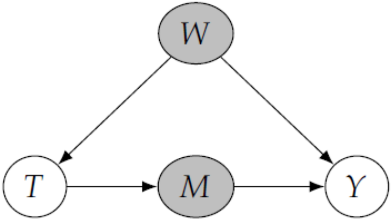

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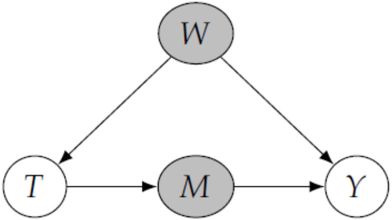

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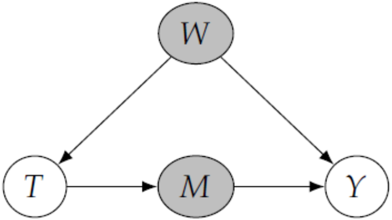

DAG Approach: Backdoor Adjustment

- Backdoor Adjustment Theorem

Theorem (Backdoor Adjustment)

If W satisfies the backdoor criterion, we can identify the causal effect of T on Y by

$$P(y|do(t)) = \int_{w} P(y|t, w)P(w)$$

- W is what we usually call "control variables"
- The backdoor criterion is similar to the "selection on observables" assumption

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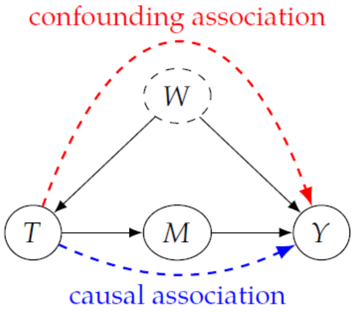
- This is totally new to economists
- Assume that we have the following DAG



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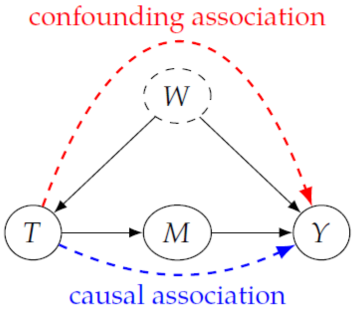

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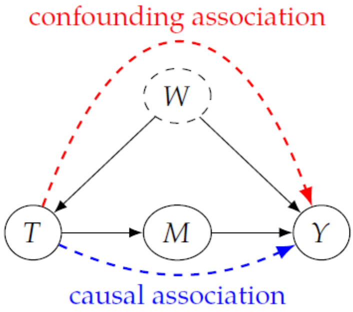

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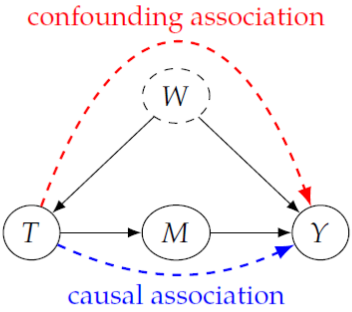

DAG Approach: Frontdoor Adjustment

- We can identify effect of T on Y in three steps
  - 1. Identify effect of T on M
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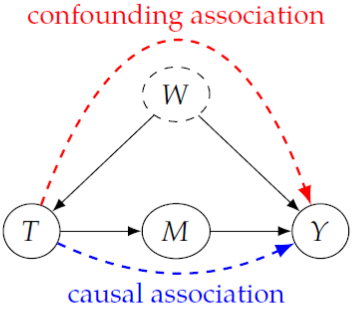

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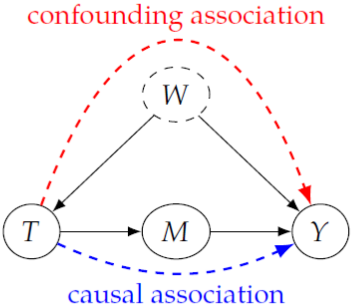

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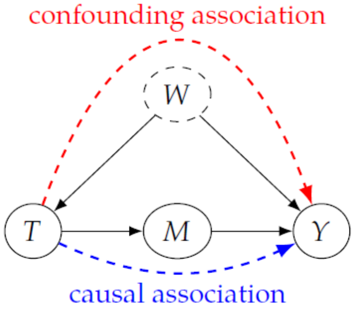

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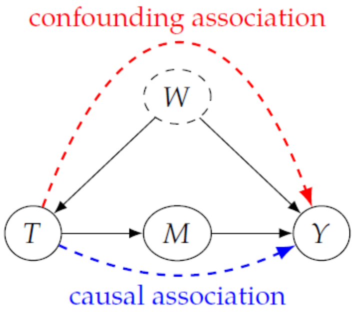

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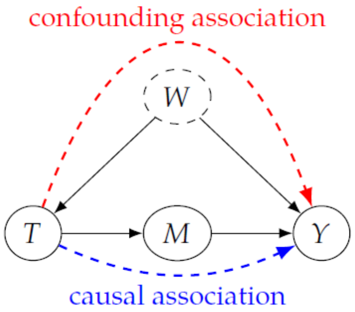

DAG Approach: Frontdoor Adjustment

Definition (Frontdoor Criterion)

A set of variables M satisfies the frontdoor criterion relative to T and Y if:

1. *M completely mediates the causal effect of T on Y;*
2. *There is no unblocked backdoor path from T to M;*
3. *All backdoor paths from M to Y are blocked by T.*

Theorem (Frontdoor Adjustment)

If T, M, Y satisfy the frontdoor criterion, then we have

$$P(y|do(t)) = \sum_{m} P(m|t) \sum_{t^{'}} P(y|m,t^{'})P(t^{'})$$

- We can identify the original treatment effect if we have a complete mediator

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- They are not necessary
- Can we find a set of necessary conditions?
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- Denote $G_{\overline{X}}$ as take graph G and then remove all incoming edges to X
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Theorem (Rules of do-calculus)

(1) *Rule 1:* $P(y|do(t), z, w) = P(y|do(t), w)$, *if* $Y \perp_{G_{\overline{T}}} Z|T, W$
(2) *Rule 2:* $P(y|do(t), do(z), w) = P(y|do(t), z, w)$, *if* $Y \perp_{G_{\overline{T}\underline{Z}}} Z|T, W$
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Theorem (Identification of Causal Effect)

A causal effect Q is identifiable in a model characterized by a graph G if there exists a finite sequence of transformations, each conforming to one of the inference rules 1, 2, or 3, that reduce Q into a standard ("do"-free) probability expression involving observed quantities.

- do-calculus is complete. You can use these three rules to identify all identifiable causal estimands.
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What are the intuitions of the three rules?

- Rule 1 (deletion of var): $P(y|do(t), z, w) = P(y|do(t), w)$, if $Y \perp_{G_{\overline{T}}} Z|T, W$
  - Erase $do(t)$, this is just an extension of d-separation under the Markov assumption
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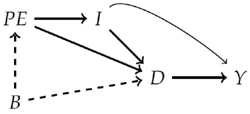
DAG Approach: An Example

- An example: College (D) return on wages (Y)
- Which variable do we need to control for?



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- I: family income
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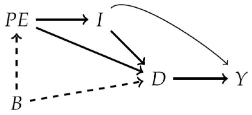

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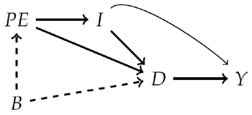

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DAG in Economics: Clarity

- Unconfoundedness

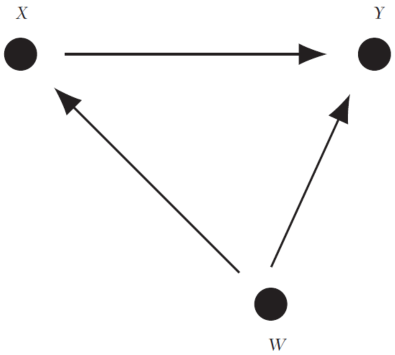

Figure 2. Unconfoundedness

DAG in Economics: Clarity

- IV strategy

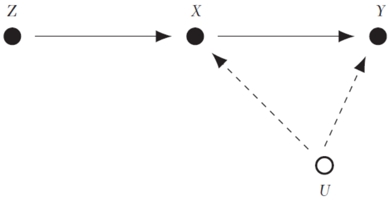

Figure 3. Instrumental Variables

DAG in Economics: Complicated Model

- An example of a complicated model

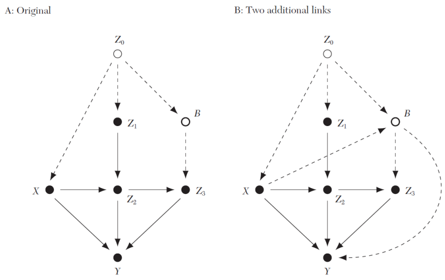

Figure 4. Two Examples of Complex DAGs

DAG in Economics: Complicated Model

- Structural Equation Modeling

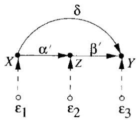

Figure 5.13 Diagram representing model M' of (5.12)–(5.14).

$$x = \varepsilon_1, \tag{5.12}$$

$$z = \alpha' x + \varepsilon_2, \tag{5.13}$$

$$y = \beta' z + \delta x + \varepsilon_3. \tag{5.14}$$

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- Imbens' concern: do we really need such huge model and SEM in econ?
- He argues that economists don't like SEM without economic meaning
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- Frontdoor adjustment can be an interesting identification strategy for economists
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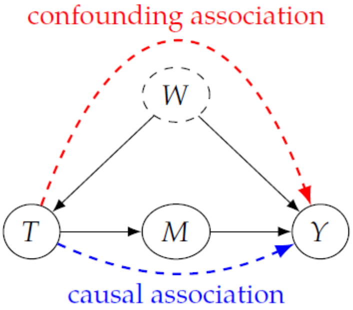

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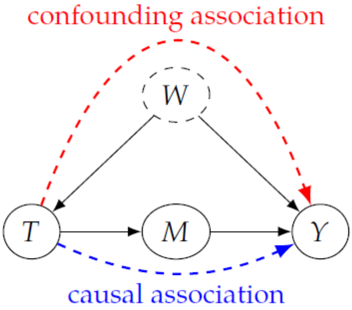

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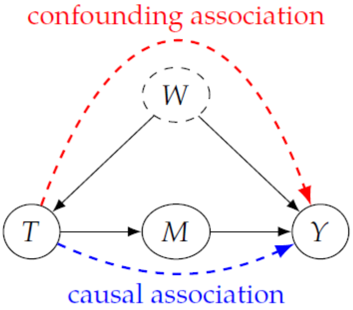

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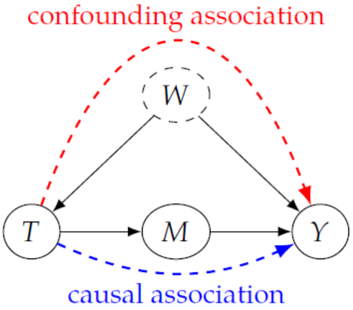

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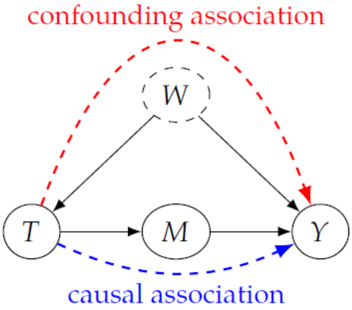

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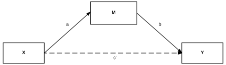

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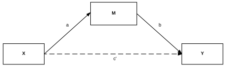

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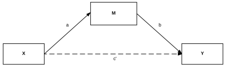

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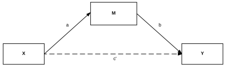

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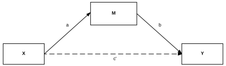

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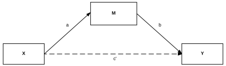

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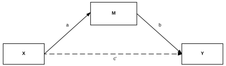

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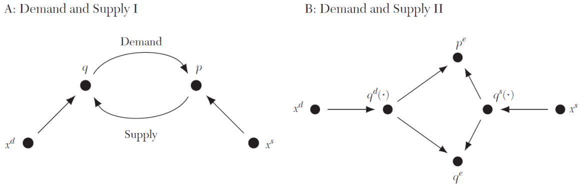

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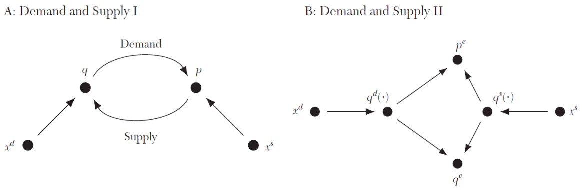

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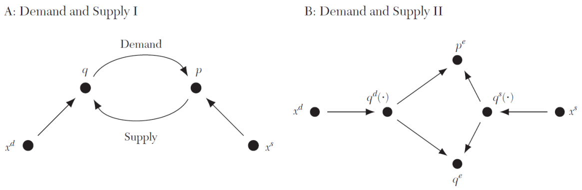

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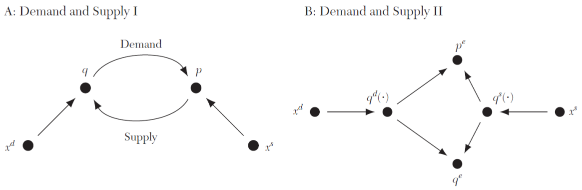

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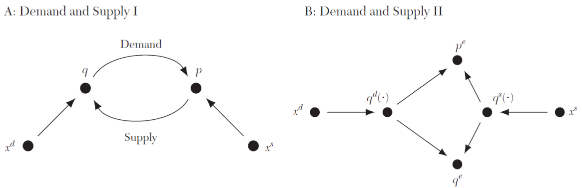

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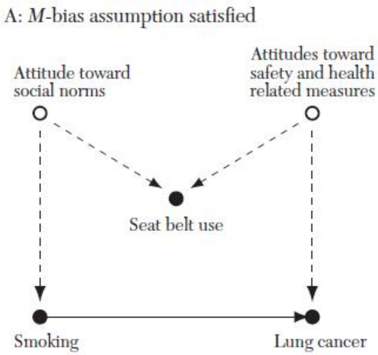

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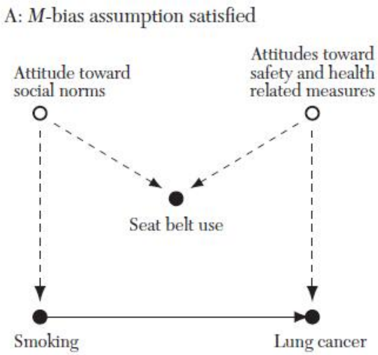

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- Sadly, in his latest version, Pinto deletes all DAG stuffs...
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Final Conclusion

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