

Frontier Topics in Empirical Economics: Week 4

Directed Acyclic Graph

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Introduction

- Causal inference is the central topic of applied economics
- We almost solely focus on potential outcome framework in Economics
- This framework is proposed by Donald Rubin (Imbens and Rubin, 2015; Rubin, 1974) and sometimes called "Rubin Causal Model"

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- Is this the only statistical framework dealing with causal inference issue?
Of course NOT.
- Graphical Model is another important method (Pearl, 2009)
- Today we are going to learn this new framework
- How it can be applied to economic research is still a *very very open question*
- Imbens wrote an interesting and critical paper on it
Imbens (2020) Potential Outcome and Directed Acyclic Graph Approaches to Causality: Relevance for Empirical Practice in Economics

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- Introduce the graphical model and the DAG framework
- Discuss the possible usage of DAG for economists: Pros and Cons
- Compare DAG and PO framework: why PO is still more popular
- An example of using DAG: Pinto (2015)
- Conclusion: How can DAG help applied economics research (open question)

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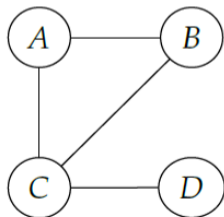
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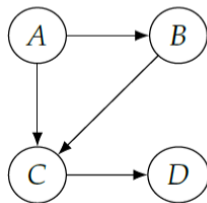
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DAG Approach: Graph

- Graph is a collection of *nodes* and *edges* that connect the nodes.
- Two nodes are called *adjacent* if they are connected by an edge.
- A directed graph's edges go out of a *parent* into a *child*.
- A *path* is any sequence of adjacent nodes, regardless of the direction of the edges. A *directed path* is a path that consists of directed edges that are all directed in the same direction.



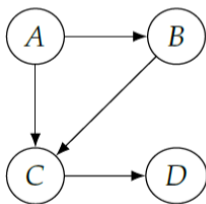
(a) Undirected Graph



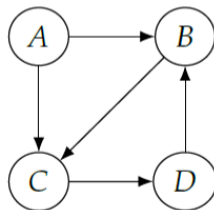
(b) Directed Graph

DAG Approach: Graph

- If there is a directed path that starts at node X and ends at node Y, then X is an ancestor of Y, and Y is a descendant of X.
- If there is no cycle in a directed graph, the graph is called a *directed acyclic graph* (DAG)



(c) Directed Graph



(d) Directed Graph with Cycle

DAG Approach: Bayesian Networks

- How to connect graphs to causal inference?
- The first step is to connect graphs to statistical relations: Bayesian Networks
- For any PDF, a Bayesian factorization can be expressed as:

$$P(x_1, x_2, \dots, x_n) = P(x_1) \prod_{i=1}^{n-1} P(x_{i+1} | x_1, \dots, x_i) \quad (1)$$

- Example: $P(x_1, x_2, x_3) = P(x_1)P(x_2|x_1)P(x_3|x_2, x_1)$
- We can simplify the model if we assume some dependency structure, e.g.
 $P(x_3|x_2, x_1) = P(x_3|x_2)$ if $x_1 \perp x_3|x_2$
- We can use a graph to represent this assumed dependency structure, system of probabilistic relations!
- A one-to-one mapping between graph G and probabilistic relations P

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DAG Approach: Bayesian Networks

Assumption (Minimality Assumption)

1. *Given its parents in the DAG, a node X is independent of all its non-descendants (Local Markov Assumption);*
2. *Adjacent nodes in the DAG are dependent (Minimal independence).*

Definition (Bayesian Network Factorization)

Given a probability distribution P and a DAG G satisfying "Minimality Assumption", P factorizes according to G by

$$P(x_1, x_2, \dots, x_n) = P(x_1) \prod_i P(x_i | pa_i)$$

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DAG Approach: Bayesian Networks

- Local Markov means that the dependence structure is "local" and "Markov"
- Minimal independence means that there is no more independence outside the graph
- Bayesian Factorization means that: If P has a causal structure as shown in G
 - X only depends on parents in the graph
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- We call " G represents P ", " G and P are compatible", " P is Markov relative to G "

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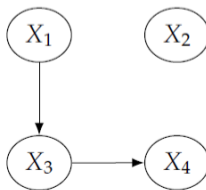
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DAG Approach: Bayesian Networks

- Let's see a simple example
- Assume that we have four variables x_1, x_2, x_3, x_4
- A full decomposition is:

$$P(x_1, x_2, x_3, x_4) = P(x_1)P(x_4|x_3, x_2, x_1)P(x_3|x_2, x_1)P(x_2|x_1) \quad (2)$$

- What if we have the following DAG showing the relation among x_1, x_2, x_3, x_4 ?

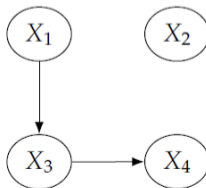


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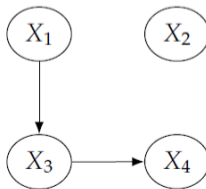


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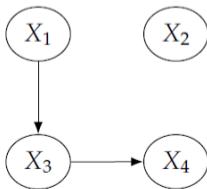


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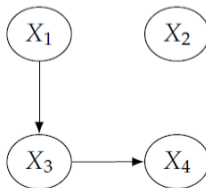


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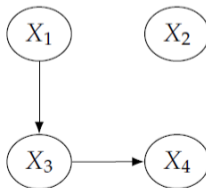


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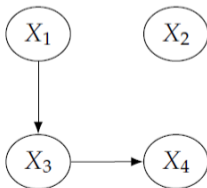


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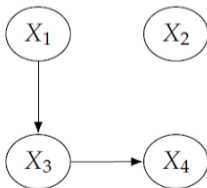


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DAG Approach: Causal Graphs

- Up until now, we consider only statistical dependencies
- What about those arrows?

In a directed graph, every parent is a direct cause of all its children.

- By adding causal edge assumption, we have this DAG to represent not only statistical dependencies, but causal relations
- Directed paths in DAGs correspond to causation
- A more mathematically rigorous definition is imposed on SEM

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- By adding causal edge assumption, we have this DAG to represent not only statistical dependencies, but causal relations
- Directed paths in DAGs correspond to causation
- A more mathematically rigorous definition is imposed on SEM

DAG Approach: Causal Graphs

- Up until now, we consider only statistical dependencies
- What about those arrows?

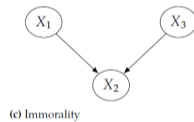
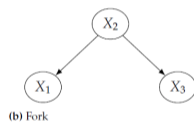
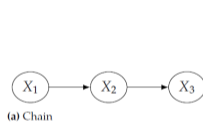
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DAG Approach: Graphical Building Blocks

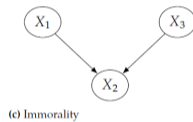
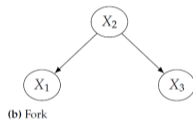
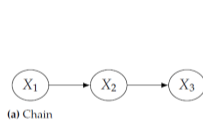
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- Flow of association is symmetric: x_1 and x_3 are associated in both chain and fork (but not immortality)
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DAG Approach: Graphical Building Blocks

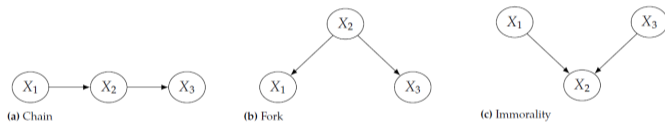
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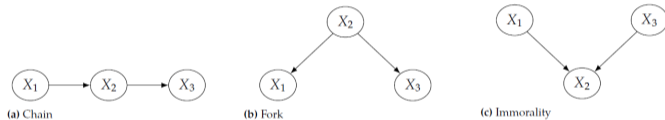
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- By conditioning on variable x_2 , we can block the flow of association in chains and forks

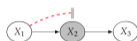


Figure 3.14: Chain with association blocked by conditioning on X_2 .

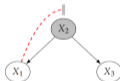


Figure 3.15: Fork with association blocked by conditioning on X_2 .

- We can show that with this graph:

$$P(x_1, x_3 | x_2) = P(x_1 | x_2)P(x_3 | x_2) \quad (4)$$

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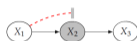


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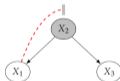


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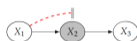


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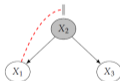


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- Things can be different in immorality
- We call X_2 , the child of a immorality, as a *collider*

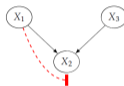


Figure 3.16: Immorality with association blocked by a collider.

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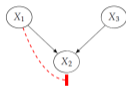


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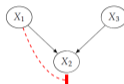


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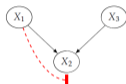


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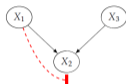


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- Controlling for post-determined variables!
- A simple example: x_1 is good-looking, x_2 is kindness, x_3 is marriage availability
- Conditional on $x_3 = 1$, you will see negative relation between x_1 and x_2 !
- Well-known as bad control problem in econometrics

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DAG Approach: Blocked Path and d-separation

Definition (Blocked Path)

A path between X and Y is blocked by a conditioning set Z if *either* of the following is true:

1. Along the path, there is a chain $\rightarrow W \rightarrow$ or a fork $\leftarrow W \rightarrow$ where $W \in Z$;
2. There is a collider W that both itself and its descendants are not conditioned on in Z ;

- Association flows along unblocked paths, does NOT flow along blocked paths!

Two sets of nodes X and Y are d-separated by a set of nodes Z if all of the paths between nodes in X and nodes in Y are blocked by Z .

- d-separation means conditional independence!
- All association flows between X and Y are blocked by Z

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DAG Approach: Blocked Path and d-separation

- Theorem 1.2.4, 1.2.5 in Pearl (2009), Theorem 3.1 in Neal (2020)

If X and Y are d-separated in a DAG G conditional on Z , then X and Y are independent conditional on Z in every distribution compatible with G .

$$X \perp\!\!\!\perp Y | Z \text{ in } P \text{ compatible with } G$$

Conversely, if X and Y are independent conditional on Z in all P compatible with G , then X and Y are d-separated in G conditional on Z .

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- Causations flow along directed unblocked paths
- Identification: how to net causation out of associations?
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- We define operator " $do(T = t)$ " as an intervention to give the whole population treatment t
- We denote it in terms of potential outcomes as:

$$P(y|do(t)) = P(Y = y|do(T = t)) = P(Y(t) = y) \quad (6)$$

- $P(y|do(t))$ means the distribution of the potential outcome $P(Y(t) = y)$
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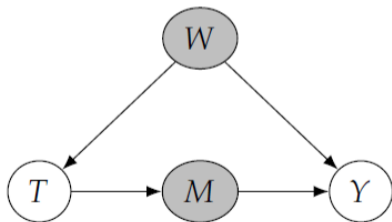
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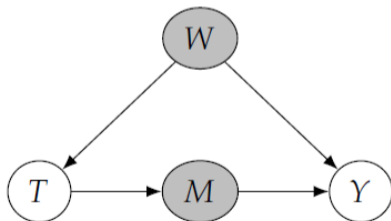
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- If some variable set W blocks all backdoor paths from T to Y and does not contain any descendants of T , we say W satisfies "the backdoor criterion"



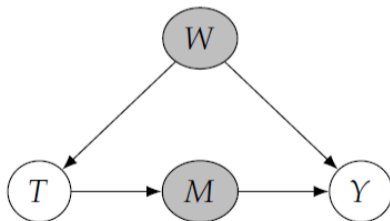
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DAG Approach: Backdoor Adjustment

- Backdoor Adjustment Theorem

If W satisfies the backdoor criterion, we can identify the causal effect of X on Y by

$$P(Y|X) = \int P(Y|X,W)P(W)$$

- W is what we usually call "control variables"
- The backdoor criterion is similar to the "selection on observables" assumption

DAG Approach: Backdoor Adjustment

- Backdoor Adjustment Theorem

Theorem (Backdoor Adjustment)

If W satisfies the backdoor criterion, we can identify the causal effect of T on Y by:

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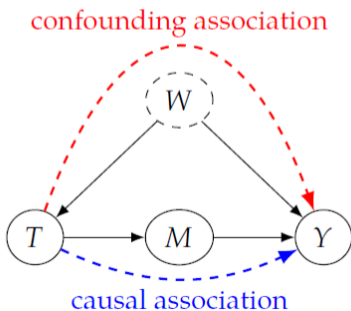
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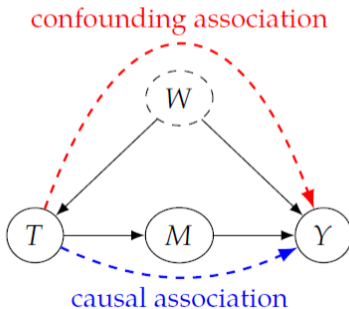
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- This is totally new to economists
- Assume that we have the following DAG



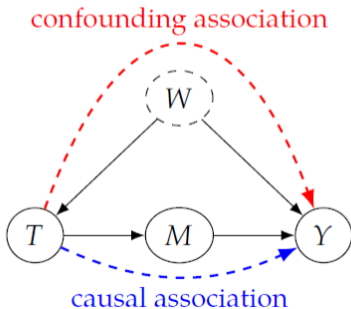
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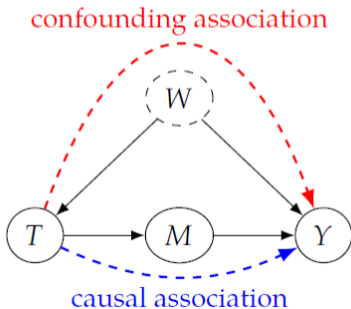
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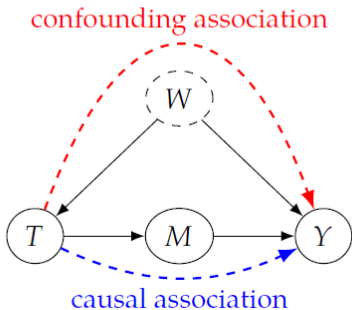
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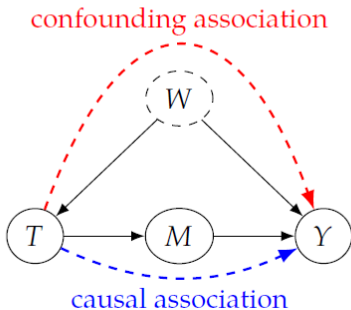
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- We can identify effect of T on Y in three steps
 1. Identify effect of T on M
 2. Identify effect of M on Y (ignoring T)
 3. Combine step 1 and 2



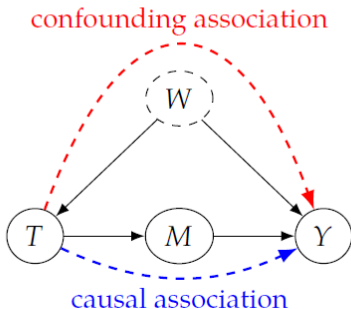
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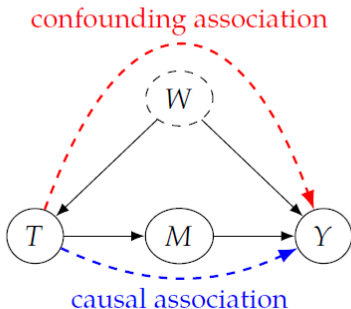
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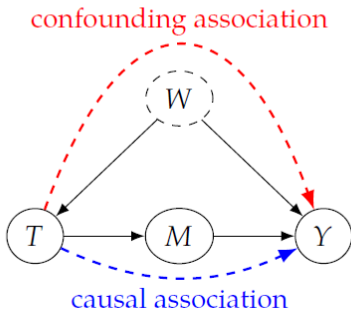
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DAG Approach: Frontdoor Adjustment

Definition (Frontdoor Criterion)

A set of variables M satisfies the frontdoor criterion relative to T and Y if:

1. M completely mediates the causal effect of T on Y ;
2. There is no unblocked backdoor path from T to M ;
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Theorem (Frontdoor Adjustment)

If T , M , Y satisfy the frontdoor criterion, then we have

$$P(y|do(t)) = \sum_m P(m|t) \sum_{t'} P(y|m, t') P(t')$$

- We can identify the original treatment effect if we have a complete mediator

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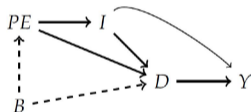
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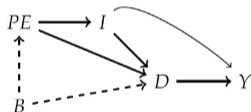
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- ▶ I : family income
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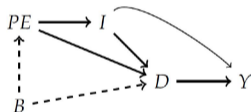
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Especially if you are interested in mediation/substitution
 - Machinery developed in DAG (do-calculus) allows researchers to investigate causal queries in a systematic way
Especially for complex models with large number of variables

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DAG in Economics

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- 1. PO framework has several features fitting applied econ better
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DAG in Economics: Clarity

- Unconfoundedness

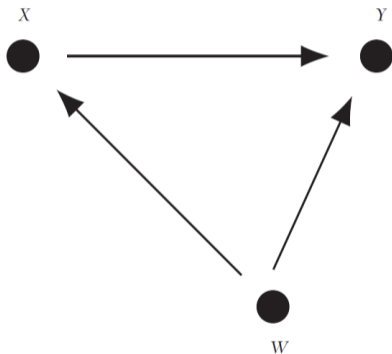


Figure 2. Unconfoundedness

DAG in Economics: Clarity

- IV strategy

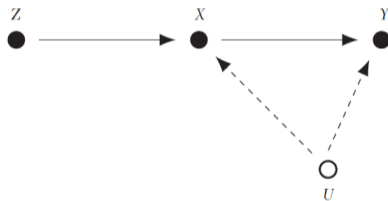
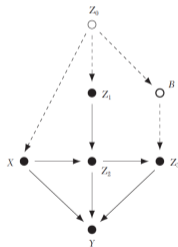


Figure 3. Instrumental Variables

DAG in Economics: Complicated Model

- An example of a complicated model

A: Original



B: Two additional links

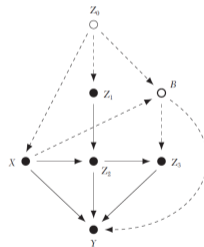


Figure 4. Two Examples of Complex DAGs

DAG in Economics: Complicated Model

■ Structural Equation Modeling

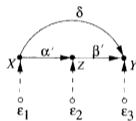


Figure 5.13 Diagram representing model M' of (5.12)–(5.14).

$$x = \varepsilon_1, \quad (5.12)$$

$$z = \alpha'x + \varepsilon_2, \quad (5.13)$$

$$y = \beta'z + \delta x + \varepsilon_3. \quad (5.14)$$

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- He argues that economists don't like SEM without economic meaning
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DAG in Economics: Frontdoor Criterion

- Frontdoor adjustment can be an interesting identification strategy for economists
- It relies on the existence of a complete mediator
- How to apply this method to economics is still an open question
- Too hard to find such a DAG in real life

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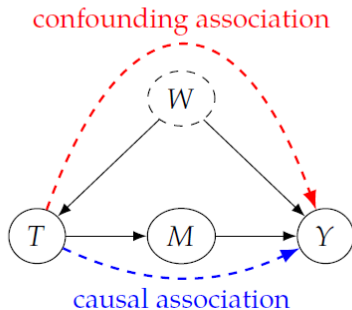
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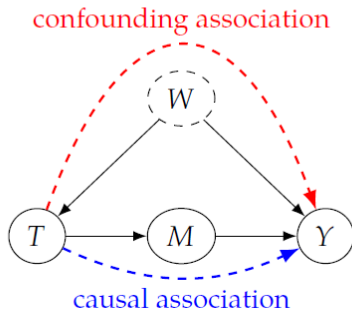
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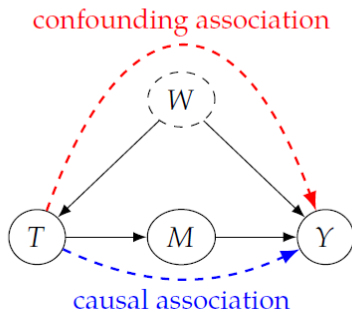
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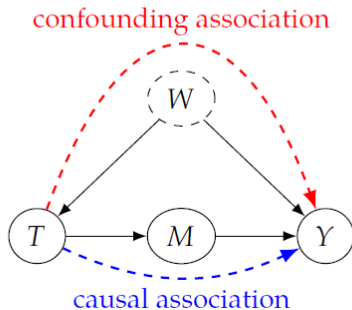
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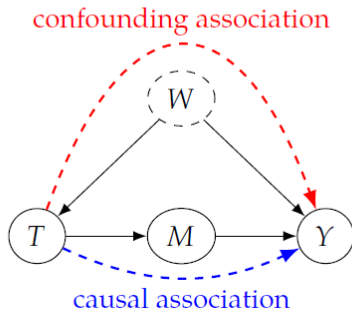
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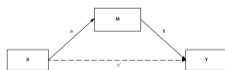
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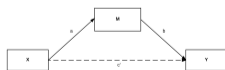
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- DAG may shed lights on identifying mediation effect
- The question remains: we need to impose strong causal structure assumption
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- Mediation effect test forces you to admit a very simple causal structure just to implement an on-the-shelf test
- This is a typical behavior of regression monkey
- DAG allows you to "have a causal structure" based on your economic context



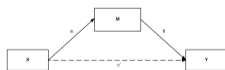
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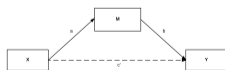
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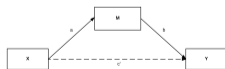
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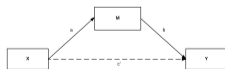
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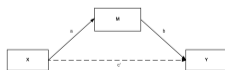
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- DAG is unable to clearly express some parts of the IV method
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 - The LATE theorem is not easily derived in a DAG approach
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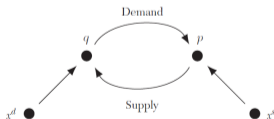
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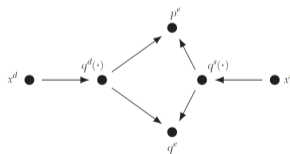
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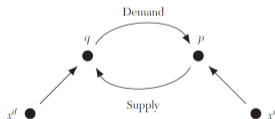
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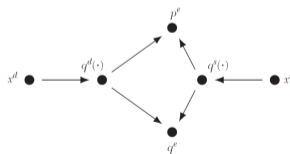
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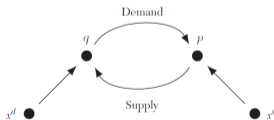
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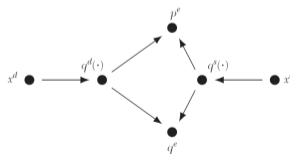
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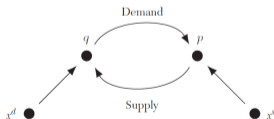
B: Demand and Supply II



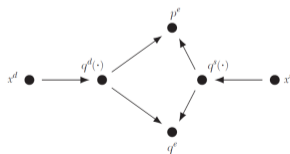
DAG in Economics: Simultaneity

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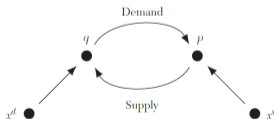
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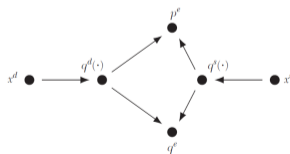
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- Angrist only gives a rule-of-thumb: control variables happening after treatment

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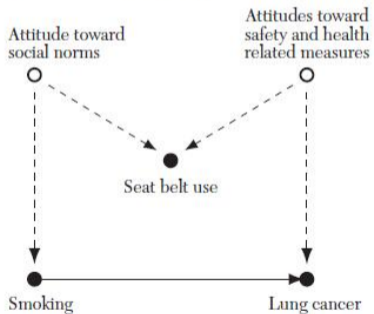
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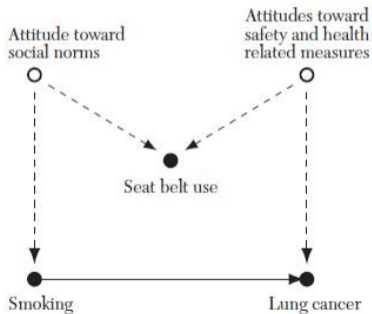
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Final Conclusion

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